

Dynamic Logic of Communicating Hybrid Programs

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Abstract. This paper presents a dynamic logic $d\mathcal{L}_{\text{CHP}}$ for compositional deductive verification of communicating hybrid programs (CHPs). CHPs go beyond the traditional mixed discrete and continuous dynamics of hybrid systems by adding CSP-style operators for communication and parallelism. A compositional proof calculus is presented that modularly verifies CHPs including their parallel compositions from proofs of their subprograms by assumption-commitment reasoning in dynamic logic. Unlike Hoare-style assumption-commitments, $d\mathcal{L}_{\text{CHP}}$ supports intuitive symbolic execution via recorder variables for communication primitives. Since $d\mathcal{L}_{\text{CHP}}$ is a conservative extension of differential dynamic logic $d\mathcal{L}$, it can be used soundly along with the $d\mathcal{L}$ proof calculus and $d\mathcal{L}$'s complete axiomatization for differential equation invariants.

Keywords: Compositional verification · Hybrid systems · Parallel programs · Differential dynamic logic · Assumption-commitment reasoning · CSP

1 Introduction

Their prevalence in safety-critical applications and ample technical subtleties make both cyber-physical systems (CPS) verification [1, 2, 12, 30] and parallel program verification [3, 20, 27, 38] important challenges. CPS verification complexity stems from subtle interactions of their discrete control decisions and differential equations. Parallel program verification complexity comes from intricate interactions caused by synchronization via state or communication interdependencies between parallel components. But their combination becomes intrinsically complicated because parallel CPS are *always* interdependent as they *always* synchronize implicitly by sharing the same global time. Moreover, many real-world CPS have heterogeneous subsystems whose controllers do not operate in lock-step, and their communication is potentially unreliable (see Fig. 1). Unlike hybrid systems model checking [2, 4, 8, 23]—which needs to compute products of communicating parallel automata of significant size even when reducing the size of the subautomata—deductive approaches can be truly compositional. Existing deductive verification approaches, however, fail to properly tackle the above

challenges, since they are restricted to homogeneous subsystems [32], specific component shapes and interaction patterns [19, 22, 26], do not support symbolic execution [10, 21, 41], or are non-compositional [10, 21, 41], where even explicit attempts on compositionality [10, 41] turn out to be non-compositional again.

Neither compositionality to tame complexity by reasoning separately about discrete, continuous, and communication behavior, nor generality in expressing models, nor symbolic execution to boost the feasibility of deductive reasoning are dispensable for a practical approach. Thus, to tackle all three of these challenges, this paper presents $d\mathcal{L}_{\text{CHP}}$, a *dynamic logic for communicating hybrid programs (CHPs)*, that extends differential dynamic logic $d\mathcal{L}$ for hybrid programs of differential equations [30, 31, 33, 34] with CSP-style operators for parallel composition and message passing communication [14, 15] as well as assumption-commitment (ac) reasoning [25, 43]. Parallel CHPs cannot share state but can communicate. They run synchronously in global time and their communication is instantaneous.

There are two fundamental approaches to reasoning about parallelism. Parallelism can either be understood via unfolding its explicit operational semantics, or via its denotational semantics implicitly characterizing parallel behavior by matching behavior of the subprograms on their respective part of the state. Verification based on the operational semantics unrolls the exponentially many possible interleavings, as in hybrid automata [2] or hybrid CSP [10, 41]. Such approaches admit superficial mitigation of the state space explosion but always resorts to product automata after reducing the size of the subautomata [4, 8, 13], or merely postpone the burden of reasoning about exponentially many trace interleavings [10, 41] since it reveals the internal structure of subprograms [41]. In contrast, verification based on the denotational semantics is compositional for discrete programs with ac-reasoning [43, 44], *if* the semantics is compositional and aligns well with the intended reasoning. This paper generalizes ac-reasoning for the purpose of integration with a compositional hybrid systems logic [30, 31, 33, 34].

Our central contribution is a sound *compositional* proof calculus for $d\mathcal{L}_{\text{CHP}}$. For compositional verification of parallel communication, it embeds ac-reasoning into dynamic logic via the explicit ac-modality $[\alpha]_{\{A, C\}}\psi$. The ac-modality expresses that for all runs of α whose incoming communication meets assumption A the outgoing communication fulfills commitment C and that ψ holds in the final state. Formulas A and C specify the communication behavior of α , and

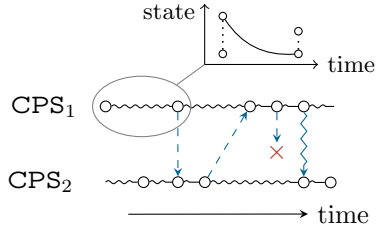


Fig. 1: The interaction of CPS_1 and CPS_2 has (delayed) ($-->$), lossy ($-->\times$), and noisy (\rightsquigarrow) communication. Discrete change (\circ) is independent of discrete change in parallel unless synchronization takes place. Continuous evolution (\rightsquigarrow) is not interrupted by parallel discrete behavior.

enable parallel composition if they mutually agree for the contracts of subprograms. Crucially, these formulas directly interface the communication history via history variables such that communication can remain implicit in the parallel composition axiom just as in the underlying denotational semantics. Since we prove that $d\mathcal{L}_{\text{CHP}}$ is a conservative extension of $d\mathcal{L}$, it inherits $d\mathcal{L}$'s complete axiomatization of differential equation invariants [35].

Unlike approaches built on Hoare-logics [10, 21, 41], our calculus supports statement-by-statement symbolic execution instead of executing them backward. Since executing communication primitives extends the former history, the new history state requires a fresh name. As a consequence, it is unsound to adopt Hoare-style ac-reasoning [43] verbatim with a distinguished variable for the communication history. Instead, $d\mathcal{L}_{\text{CHP}}$ reconciles ac-reasoning and symbolic execution via recorder variables whose evolution is maintainable, as done by our communication axiom along the way.

In summary, we provide the first truly *compositional* approach to the verification of communicating parallel hybrid system models and prove its soundness. Our logical treatment separates the essentials of discrete, continuous, and communication reasoning into axioms that are simple and modular compared to earlier approaches [21, 41]. Even though the technical development is challenging because of a prefix-closed dynamic semantics, a subtle static semantics due to the global time and recorder variables, and the mutual dependency of formulas and programs in dynamic logic it remains finally under the hood. We demonstrate the flexibility of $d\mathcal{L}_{\text{CHP}}$ and its proof calculus with an example in autonomous driving considering the challenge of lossy communication, where a *follower* and *leader* car communicate to form a convoy.

2 Dynamic Logic of Communicating Hybrid Programs

We introduce $d\mathcal{L}_{\text{CHP}}$, a *dynamic logic* to reason about *communicating hybrid programs* (CHPs). CHPs combine $d\mathcal{L}$'s hybrid programs [30, 33, 34] with communication primitives and a parallel operator similar to CSP [14, 15]. On the logical side, $d\mathcal{L}_{\text{CHP}}$ introduces ac-reasoning [25, 43, 44] into the dynamic logic setup [11] of $d\mathcal{L}$, allowing compositional reasoning about communication in a way that preserves symbolic execution as an intuitive reasoning principle.

2.1 Syntax

The syntax uses channel names Ω and variables $V = V_{\mathbb{R}} \cup V_{\mathbb{N}} \cup V_{\mathcal{T}}$ with pairwise disjoint sets of real variables $V_{\mathbb{R}}$, integer variables $V_{\mathbb{N}}$, and trace variables $V_{\mathcal{T}}$. The variable $\mu \in V_{\mathbb{R}}$ is designated to reflect the global time. By convention $x, y, t \in V_{\mathbb{R}}$, $n, n_i \in V_{\mathbb{N}}$, $h, h_i \in V_{\mathcal{T}}$, $ch, ch_i \in \Omega$, and $z, z_i \in V$. Notions $FV(\cdot)$ of free and $BV(\cdot)$ of bound variables in formulas and programs are defined as usual by simultaneous induction with the syntax (see Appendix B). $V(\cdot) = FV(\cdot) \cup BV(\cdot)$ is the set of all variables, whether read or written.

All real variables $V_{\mathbb{R}}$ can be read *and* written in programs but the global time μ is *not meant* to be written manually.⁴ Instead, the built-in evolution of μ with every continuous behavior makes it represent the global flow of time. Trace variables are bound by programs when they record communication. For a program α , we call the remaining set $(BV(\alpha) \cap V_{\mathbb{R}}) \setminus \{\mu\}$ the state of α and say that α operates over a real state. In parallel compositions $\alpha \parallel \beta$, the programs α and β may communicate explicitly but do *not* share state.

The logical language of $d\mathcal{L}_{\text{CHP}}$ features trace algebra to reason about communication behavior following the approach of Zwiers *et al.* [42,44]. During symbolic program execution, communication events are collected syntactically in logical trace variables that were explicitly designated to record the history. This way, the communication behavior of a program can be specified using recorder variables as interface. In analogy to a distinguished history variable, this interface is crucial to obtain a compositional proof rule for parallel composition [17,38,44].

Definition 1 (Terms). *Real terms $\text{Trm}_{\mathbb{R}}(V, \Omega)$, integer terms $\text{Trm}_{\mathbb{N}}(V, \Omega)$, channel terms $\text{Trm}_{\Omega}(V, \Omega)$, and trace terms $\text{Trm}_{\mathcal{T}}(V, \Omega)$ are defined by the grammar below, where $c \in \mathbb{Q}$ is a rational constant, $ch \in \Omega$ a channel name, $\theta_1, \theta_2 \in \mathbb{Q}[V_{\mathbb{R}}]$ are polynomials in $V_{\mathbb{R}}$, and $C \subseteq \Omega$ is a finite set of channel names. The set of all terms is denoted by $\text{Trm}(V, \Omega)$.*

$$\begin{aligned} \text{Trm}_{\mathbb{R}}(V, \Omega) : & \quad \eta_1, \eta_2 ::= x \mid c \mid \eta_1 + \eta_2 \mid \eta_1 \cdot \eta_2 \mid \mathbf{val}(te[ie]) \mid \mathbf{time}(te[ie]) \\ \text{Trm}_{\mathbb{N}}(V, \Omega) : & \quad ie_1, ie_2 ::= n \mid 0 \mid 1 \mid ie_1 + ie_2 \mid |te| \\ \text{Trm}_{\Omega}(V, \Omega) : & \quad ce_1, ce_2 ::= ch \mid \mathbf{chan}(te[ie]) \\ \text{Trm}_{\mathcal{T}}(V, \Omega) : & \quad te_1, te_2 ::= h \mid \epsilon \mid \langle ch, \theta_1, \theta_2 \rangle \mid te_1 \cdot te_2 \mid te \downarrow C \end{aligned}$$

Real terms $\text{Trm}_{\mathbb{R}}(V, \Omega)$ are formed by arithmetic operators, variables x (including μ), and rational constants c . Additionally, $\mathbf{val}(te[ie])$ accesses the value and $\mathbf{time}(te[ie])$ the timestamp of the ie -th communication in trace te . In CHPs only polynomials $\theta \in \mathbb{Q}[V_{\mathbb{R}}] \subset \text{Trm}_{\mathbb{R}}(V, \Omega)$ in $V_{\mathbb{R}}$ over rational coefficients occur, i.e., without trace terms, since CHPs operate over a real state. By convention, θ, θ_i denote terms from $\mathbb{Q}[V_{\mathbb{R}}]$.

For integers, we use Presburger arithmetic (no multiplication) since it is decidable [37] and sufficient for our purposes.⁵ The integer term $|te|$ denotes the length of trace te . In analogy to $\mathbf{val}(te[ie])$, the term $\mathbf{chan}(te[ie])$ accesses the channel name of the ie -th communication in trace te . The trace term ϵ represents empty communication, $te_1 \cdot te_2$ is the concatenation of trace terms te_1 and te_2 , and $te \downarrow C$ the projection of te onto the set of channel names $C \subseteq \Omega$. The tuple $\langle ch, \theta_1, \theta_2 \rangle$ represents communication of θ_1 along channel ch at time θ_2 , where $\theta_1, \theta_2 \in \mathbb{Q}[V_{\mathbb{R}}]$ since communication is between programs over a real state.

A trace variable h refers to a sequence of communication events. By symbolic execution, proofs collect communication items in trace variables designated

⁴ Programs writing the global time μ manually are likely to be meaningless such as $\mu := 0 \parallel \mu := 1$ that sets μ to 0 and 1 in parallel, which fortunately has no runs.

⁵ Presburger arithmetic is the subset of $\text{Trm}_{\mathbb{N}}(V, \Omega)$ without length computations $|te|$.

to record the history. Then, communication behavior is specified against these recorder variables using projections onto the channels of interest (see Example 1 below). This allows specifications to hide the internal structure of programs leading to compositional reasoning in the presence of communication [17, 38, 44].

Notation. We write $\text{val}(te)$ to abbreviate $\text{val}(te[|te| - 1])$, i.e., access to the value of the last item on trace te . Likewise, we use $\text{time}(te)$ and $\text{chan}(te)$.

Example 1. The formula $|h \downarrow \text{ch}| > 0 \rightarrow \text{val}(h \downarrow \text{ch}) > 0$ expresses that the last value sent along channel ch recorded by h is positive. The precondition $|h \downarrow \text{ch}| > 0$ ensures that the value is accessed only for a non-empty history.

Definition 2 (Communicating hybrid programs). *The set $\text{CHP}(V, \Omega)$ of communicating hybrid programs is defined by the grammar below, where $x \in V_{\mathbb{R}}$ for x, x' , and $\theta \in \mathbb{Q}[V_{\mathbb{R}}]$ is a polynomial in $V_{\mathbb{R}}$, and $\chi \in \text{FOI}_{\mathbb{R}}(V_{\mathbb{R}})$ is a formula of first-order real-arithmetic. In the parallel composition $\alpha \parallel \beta$, the constituents must not share state, i.e., $V(\alpha) \cap \text{BV}(\beta) = V(\beta) \cap \text{BV}(\alpha) \subseteq \{\mu\} \cup V_{\mathcal{T}}$.*

$$\begin{aligned} \alpha, \beta ::= & x := \theta \mid x := * \mid \{x' = \theta \ \& \ \chi\} \mid ?\chi \mid \alpha; \beta \mid \alpha \cup \beta \mid \alpha^* && \text{(standard } d\mathcal{L}\text{)} \\ & \text{ch}(h)! \theta \mid \text{ch}(h)?x \mid \alpha \parallel \beta && \text{(CSP extension)} \end{aligned}$$

The statement $x := \theta$ instantly changes x to θ and nondeterministic assignment $x := *$ sets x to an arbitrary value. Assignment to the global time μ is only meant to be used by axioms. As in $d\mathcal{L}$ [30], continuous evolution $\{x' = \theta \ \& \ \chi\}$ follows the differential equation $x' = \theta$ for a nondeterministically chosen duration but only as long as the domain constraint χ is fulfilled.

In $d\mathcal{L}_{\text{CHP}}$, the global time μ always evolves during a continuous evolution according to $\mu' = 1$ even if it does not occur syntactically. Hence, an evolution $\mu' = \theta$ is considered ill-formed if $\theta \neq 1$ just like an ODE $x' = 1, x' = 2$ is considered ill-formed. Since programs operate over a real state, terms $\theta \in \mathbb{Q}[V_{\mathbb{R}}]$ are polynomials in $V_{\mathbb{R}}$ and $\chi \in \text{FOI}_{\mathbb{R}}(V_{\mathbb{R}})$ is a formula of first-order real-arithmetic.

The test $?\chi$ passes if formula χ is satisfied. Otherwise, execution is aborted. The sequential composition $\alpha; \beta$ executes β after α . The choice $\alpha \cup \beta$ follows α or β nondeterministically, and α^* repeats α zero or more times.

Communication and parallelism are inspired by CSP [14]. The primitive $\text{ch}(h)! \theta$ sends the value of term θ along channel ch and $\text{ch}(h)?x$ receives a value from ch binding it to variable x . For both statements, h is the trace variable designated to record the communication. In an ongoing symbolic execution, this variable can be renamed to refer to the variable keeping the most recent communication history. In system models, all recorder variables are meant to be the same. In this case, we also write $\text{ch}! \theta$ and $\text{ch}?x$ instead of $\text{ch}(h)! \theta$ and $\text{ch}(h)?x$.

Finally, $\alpha \parallel \beta$ executes α and β in parallel for equal duration, i.e., their final states agree on the value of the global time μ . If μ is not manipulated manually, its increase equals the duration of continuous behavior. As in CSP, α and β can perform synchronous message passing but cannot share state.⁶ All programs

⁶ The global time μ and trace variables $V_{\mathcal{T}}$ are not considered as program state.

participating in communication over a channel must agree on all communication along that channel and share the same values and recorder variables at the same time, i.e., message passing does not consume time. Since shared recorder variables agree on the communication for all subprograms, they provide the interface that allows for decomposition of parallel behavior. If the recorders are not the same as in $\text{ch}(h_0)! \theta \parallel \text{ch}(h_1)?x$, there are no runs. The need for matching recorders in parallel composition must not be confused with renaming of the history by symbolic execution along the sequential structure of programs.

As usual in CSP [14], the syntax does not enforce unidirectional communication such that several programs may send and receive on the same channel at the same time as long as they agree on the recorder variables and values. For example, $\text{ch}(h)?x \parallel \text{ch}(h)?y$ is a well-formed program. Its semantics has all runs where x and y receive the same values. Likewise, $\text{ch}(h)! \theta_1 \parallel \dots \parallel \text{ch}(h)! \theta_n$ has terminating runs if the values of all θ_i agree. Consequently, a receive statement $\text{ch}(h)?x$ can be replaced with the semantical equivalent $x := *; \text{ch}(h)!x$.

Notation. As usual $\text{if}(\varphi) \{\alpha\}$ is short for $(?\varphi; \alpha) \cup ?\neg\varphi$.

Example 2. Fig. 2 models two cars in a convoy safely adjusting their speed. From time to time, the **leader** changes its speed v_l in the range 0 to V and notifies the **follower** about the change. The communication, however, is lossy ($\text{vel}!v_l \cup \text{skip}$). Sending position updates by **updt** succeeds at least every ϵ time units. On such an update, the **follower's** controller **dist** awakes. If the distance d fell below ϵV , the **follower** slows down to avoid collision before the next position update.

Regularly, the **follower** adopts the speed update in **velo**, but crucially refuses to do so if the last known distance was not safe ($d > \epsilon V$). If the **leader** could overwrite the **follower's** speed, it could cause a future collision (see Fig. 3 below) even though obeying would be perfectly fine at the moment. This is because a subsequent notification of the leader slowing down could be lost.

$$\begin{array}{ll}
\text{velo} \equiv \text{vel}?v_{tar}; \text{if}(d > \epsilon V) v_f := v_{tar} & \chi_{vel} \equiv 0 \leq v_l \leq V \\
\text{dist} \equiv \text{pos}?m; d := m - x_f; & \text{noti} \equiv v_l := *; ?\chi_{vel}; (\text{vel}!v_l \cup \text{skip}) \\
\text{if}(d \leq \epsilon V) \{v_f := *; ?0 \leq v_f < d/\epsilon\} & \text{updt} \equiv \text{pos}!x_l; w := 0 \\
\text{plant}_f \equiv \{x'_f = v_f\} & \text{plant}_l \equiv \{w' = 1, x'_l = v_l \ \& \ w \leq \epsilon\} \\
\text{follower} \equiv ((\text{velo} \cup \text{dist}); \text{plant}_f)^* & \text{leader} \equiv ((\text{noti} \cup \text{updt}); \text{plant}_l)^*
\end{array}$$

Fig. 2: Models of two moving cars (**follower** and **leader**) intended to form the convoy $\text{follower} \parallel \text{leader}$ by parallel composition, communicating target speed.

Definition 3 (Formulas). *The set of $d\mathcal{L}_{\text{CHP}}$ formulas $\text{Fml}(V, \Omega)$ is defined by the following grammar, where $z \in V$, terms $e_1, e_2 \in \text{Trm}(V, \Omega)$ are of equal sort, $\eta_i \in \text{Trm}_{\mathbb{R}}(V, \Omega)$, $ie_i \in \text{Trm}_{\mathbb{N}}(V, \Omega)$, $te_i \in \text{Trm}_{\mathcal{T}}(V, \Omega)$, and the ac-formulas **A** and **C** do not refer to state and time of α , i.e., $(FV(\mathbf{A}) \cup FV(\mathbf{C})) \cap BV(\alpha) \subseteq V_{\mathcal{T}}$.*

$$\begin{aligned}
\varphi, \psi, \mathbf{A}, \mathbf{C} ::= & e_1 = e_2 \mid \eta_1 \geq \eta_2 \mid ie_1 \geq ie_2 \mid te_1 \preceq te_2 \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \\
& \varphi \rightarrow \psi \mid \forall z \varphi \mid \exists z \varphi \mid [\alpha]\psi \mid [\alpha]_{\{\mathbf{A}, \mathbf{C}\}}\psi
\end{aligned}$$

For specifying the (communication) behavior of CHPs, we combine first-order dynamic logic [11] with ac-reasoning [43]. Equality is defined on each sort of terms. On real and integer terms, \geq has the usual meaning. On trace terms, $te_1 \preceq te_2$ means that te_1 is a prefix of te_2 . There is no order on channel terms. Quantified variables $z \in V$ are of arbitrary sort. Since our primary interest is safety, we omit the dynamic modality $\langle \alpha \rangle \psi$ and give no dual $\langle \alpha \rangle_{\{A,C\}} \psi$ for $[\alpha]_{\{A,C\}} \psi$.

Besides the dynamic modality $[\alpha] \psi$, $d\mathcal{L}_{\text{CHP}}$ prominently features the ac-box $[\alpha]_{\{A,C\}} \psi$ that reshapes Hoare-style ac-reasoning [43] into the modal approach of dynamic logic. In an ac-contract $\varphi \rightarrow [\alpha]_{\{A,C\}} \psi$, assumption **A** and commitment **C** specify α 's communication behavior along the interface of recorder variables but without access to α 's state and time as required by $(FV(\mathbf{A}) \cup FV(\mathbf{C})) \cap BV(\alpha) \subseteq V_{\mathcal{T}}$, whereas formulas φ and ψ act as pre- and postcondition as usual. Formula $[\alpha]_{\{A,C\}} \psi$ promises that **C** holds after each communication event of α assuming **A** held before each event. Moreover, if the program terminated *and* **A** held before and after each communication event, the final state satisfies ψ .

Example 3. The safety condition about **follower** and **leader** below expresses: If they start driving with a distance of at least d and a speed $\leq d/\epsilon$ that prevents the **follower** from reaching the **leader** within ϵ time units, then the cars do never collide. Section 3 shows a proof of this formula.

$$\epsilon \geq 0 \wedge 0 \leq v_f \leq d/\epsilon \wedge v_f \leq V \wedge x_f + d < x_l \rightarrow [\mathbf{follower} \parallel \mathbf{leader}] x_f < x_l$$

Closed systems, where communication has an internal partner, can be specified using boxes $[\cdot] \psi$ (see Example 3) since their safety does not depend on the environment. Ac-boxes $[\cdot]_{\{A,C\}} \psi$ come into play when such systems are decomposed since the constituents **follower** and **leader** are each other's environment.

2.2 Semantics

CHPs have a denotational linear history semantics merging ideas from $d\mathcal{L}$ [30] and ac-reasoning [44] and adding synchronization in the global time. The basic domains are traces \mathcal{T} and states \mathcal{S} . A *trace* $\tau \in \mathcal{T}$ is a finite sequence (τ_1, \dots, τ_n) of communication events $\tau_i = \langle \text{ch}_i, a_i, s_i \rangle$ with channel $\text{ch}_i \in \Omega$, value $a_i \in \mathbb{R}$,

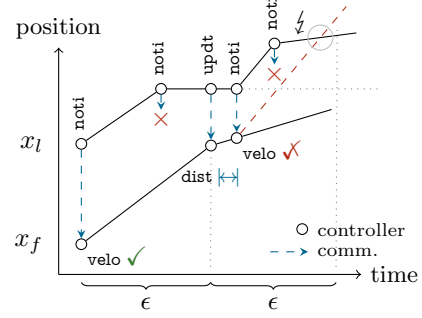


Fig. 3: Plot of example positions x_f and x_l of the cars over time. First, speed update is accepted (\checkmark). The next update is lost ($\dashrightarrow \times$). After position update (\leftarrow), the follower adjusts its speed. Crucially, it conservatively rejects the speed update (\checkmark) when a crash (ζ) with a slowing leader is possible since speed communication may fail ($\dashrightarrow \times$) until the next reliable position update is expected, see dashed trajectory (\dashrightarrow).

and timestamp $s_i \in \mathbb{R}$ that is *chronological*, i.e., $s_i \leq s_j$ for all $1 \leq i < j \leq n$. The empty trace is denoted ϵ , the concatenation of traces τ_1 and τ_2 is $\tau_1 \cdot \tau_2$, and for $C \subseteq \Omega$, the projection $\tau \downarrow C$ is the subsequence of τ consisting exactly of those $\langle \text{ch}, a, s \rangle$.⁷ By $\tau' \preceq \tau$ and $\tau' \prec \tau$, we express that τ' is a prefix or proper prefix of τ , respectively. A *recorded trace* $\tau \in \mathcal{T}_{\text{rec}}$ is a trace that has an additional recorder variable $h_i \in V_{\mathcal{T}}$ for each communication event such that $\tau_i = \langle h_i, \text{ch}_i, a_i, s_i \rangle$. Definitions for traces, such as concatenation and prefixing, accordingly apply to recorded traces. Raw traces \mathcal{T} represent trace terms in a state, whereas recorded traces originate from programs.

A *state* is a map $v : V \rightarrow \mathbb{R} \cup \mathbb{N} \cup \mathcal{T}$ that assigns a value from $\text{type}(z)$ to each variable $z \in V$, where $\text{type}(e) = \mathbb{M}$ if $e \in \text{Trm}_{\mathbb{M}}(V, \Omega)$ for $\mathbb{M} \in \{\mathbb{R}, \mathbb{N}, \mathcal{T}\}$. The *updated state* v_z^d is defined by $v_z^d = v$ on $\{z\}^c$ and $v_z^d(z) = d$. *State-trace concatenation* $v \cdot \tau$ appends recorded communication $\tau \in \mathcal{T}_{\text{rec}}$ to the corresponding trace variable in $v \in \mathcal{S}$. It is defined by $v \cdot \tau = v$ on $V_{\mathcal{T}}^c$ and $(v \cdot \tau)(h) = v(h) \cdot \tau(h)$ for all $h \in V_{\mathcal{T}}$, where $\tau(h)$ denotes the subtrace of τ consisting of the raw versions $\langle \text{ch}, a, s \rangle$ of communication events $\langle h, \text{ch}, a, s \rangle$ in τ recorded by h .

Term Semantics. The value $\llbracket e \rrbracket v \in \text{type}(e)$ of term e at state $v \in \mathcal{S}$ is according to its sort $\text{type}(e)$ (see Appendix A). The evaluation of real and integer terms is as usual. Additionally, $\text{val}(te[ie])$ evaluates to the value, $\text{time}(te[ie])$ to the timestamp, and $\text{chan}(te[ie])$ to the channel name of the ie -th communication event in te with indices from 0 to $|te| - 1$. Moreover, $|te|$ evaluates to the length of te . The evaluation of trace terms is aligned with the semantic operators on traces [42, 44], e.g., $\llbracket te \downarrow C \rrbracket v = (\llbracket te \rrbracket v) \downarrow C$ and $\llbracket \langle \text{ch}, \theta_1, \theta_2 \rangle \rrbracket v = \langle \text{ch}, \llbracket \theta_1 \rrbracket v, \llbracket \theta_2 \rrbracket v \rangle$.

Domain of Computation. The denotational semantics $\llbracket \alpha \rrbracket \subseteq \mathcal{D}$ of a CHP α has domain $\mathcal{D} = \mathcal{S} \times \mathcal{T}_{\text{rec}} \times \mathcal{S}_{\perp}$ with $\mathcal{S}_{\perp} = \mathcal{S} \cup \{\perp\}$, i.e., the observables of a CHP started from a state are communication and a final state. The marker \perp indicates an unfinished execution that either can be continued or was aborted due to a failing test. Since communication can even be observed from unfinished computations, a meaningful semantics of communicating programs is prefix-closed and total (see Def. 4 below). Totality captures that every program can at least start computation even if it aborts immediately like $?\perp$ and has not emitted communication initially. For Def. 4, we extend the prefix relation \preceq on traces \mathcal{T}_{rec} to a partial order \preceq on observable behavior $\mathcal{T}_{\text{rec}} \times \mathcal{S}_{\perp}$ expressing that (τ', w') is a prefix of (τ, w) if $((w = w'$ and $\tau = \tau')$ or $(w' = \perp$ and $\tau' \preceq \tau)$).

Definition 4 (Prefix-closedness and totality). *A set $U \subseteq \mathcal{D}$ is prefix-closed if $(v, \tau, w) \in U$ and $(\tau', w') \preceq (\tau, w)$ implies $(v, \tau', w') \in U$. The set is total if $\perp_{\mathcal{D}} \subseteq U$ with $\perp_{\mathcal{D}} = \mathcal{S} \times \{\epsilon\} \times \{\perp\}$, i.e., $(v, \epsilon, \perp) \in U$ for every $v \in \mathcal{S}$.*

⁷ We use the same operators for corresponding syntax and semantics, i.e., $te \downarrow C$ and $\tau \downarrow C$ are the projection on C for trace term te and semantic trace τ , respectively.

Program Semantics. The semantics of compound programs is compositionally defined in terms of semantical operators: For $U, M \subseteq \mathcal{D}$, we define $U_{\perp} = \{(v, \tau, \perp) \mid (v, \tau, w) \in U\}$ and $(v, \tau, w) \in U \triangleright M$ if $(v, \tau_1, u) \in U$ and $u \neq \perp$ and $(u, \tau_2, w) \in M$ exists such that $\tau = \tau_1 \cdot \tau_2$. The operator $\hat{\circ}$ is for sequential composition. For $U, M \subseteq \mathcal{D}$, we define $U \hat{\circ} M = U_{\perp} \cup (U \triangleright M)$. Semantic iteration U^m is defined by $U^0 = \mathbf{I}_{\mathcal{D}} = \perp_{\mathcal{D}} \cup (\mathcal{S} \times \{\epsilon\} \times \mathcal{S})$ and $U^{n+1} = U \hat{\circ} U^n$ for $n > 0$. Accordingly, $\alpha^0 \equiv ?\mathbf{T}$ and $\alpha^{n+1} \equiv \alpha$; α^n defines syntactic iteration.

Parallel composition $\alpha \parallel \beta$ requires that α and β have disjoint bound variables (Def. 2) except for $\{\mu\} \cup V_{\mathcal{T}}$, where they will always agree. Thus, the merged state $w_{\alpha} \oplus w_{\beta} \in \mathcal{S}_{\perp}$ for states $w_{\alpha}, w_{\beta} \in \mathcal{S}_{\perp}$ can be unambiguously determined as follows: $w_{\alpha} \oplus w_{\beta} = \perp$ if at least one of the states is \perp . Otherwise, define $(w_{\alpha} \oplus w_{\beta})(z) = w_{\alpha}(z)$ if $z \in \mathcal{BV}(\alpha)$ and $(w_{\alpha} \oplus w_{\beta})(z) = w_{\beta}(z)$ if $z \notin \mathcal{BV}(\alpha)$.⁸ For program α , the set $\mathcal{CN}(\alpha) \subseteq \Omega$ consists of all channel names occurring in α , i.e., in send $\text{ch}(h)!\theta$ and receive $\text{ch}(h)?x$ statements. The projection $\tau \downarrow \mathcal{CN}(\alpha)$ is abbreviated as $\tau \downarrow \alpha$. The *semantic parallel operator* is defined as follows for programs $\alpha, \beta \in \text{CHP}(V, \Omega)$:

$$\llbracket \alpha \parallel \beta \rrbracket = \left\{ (v, \tau, w_{\alpha} \oplus w_{\beta}) \in \mathcal{D} \mid \begin{array}{l} (v, \tau \downarrow \alpha, w_{\alpha}) \in \llbracket \alpha \rrbracket, (v, \tau \downarrow \beta, w_{\beta}) \in \llbracket \beta \rrbracket, \\ w_{\alpha}(\mu) = w_{\beta}(\mu), \tau = \tau \downarrow (\alpha \parallel \beta) \end{array} \right\}$$

Instead of computing explicit interleavings, the parallel operator \parallel characterizes the joint communication τ implicitly via any order that the subprograms can agree on. Thereby $\tau \in \mathcal{T}_{\text{rec}}$ rules out non-chronological ordering of communication events that are exclusive to either $\tau \downarrow \alpha$ or $\tau \downarrow \beta$. Moreover, by $\tau = \tau \downarrow (\alpha \parallel \beta)$, the trace τ must not contain any junk, i.e., communication not caused by α or β . Communication along joint channels of α and β must agree in its recorder variable, value, and timestamp as it occurs in $\tau \downarrow \alpha$ and $\tau \downarrow \beta$. By $w_{\alpha}(\mu) = w_{\beta}(\mu)$ both computations need to meet at the same point in global time.⁹

Definition 5 (Program semantics). *The semantics $\llbracket \alpha \rrbracket \subseteq \mathcal{D}$ of a program $\alpha \in \text{CHP}(V, \Omega)$ is inductively defined as follows, where $\perp_{\mathcal{D}} = \mathcal{S} \times \{\epsilon\} \times \{\perp\}$ and \models is the satisfaction relation for formulas (Def. 6):*

$$\begin{aligned} \llbracket x := \theta \rrbracket &= \perp_{\mathcal{D}} \cup \{(v, \epsilon, w) \mid w = v_x^{\llbracket \theta \rrbracket v}\} \\ \llbracket x := * \rrbracket &= \perp_{\mathcal{D}} \cup \{(v, \epsilon, w) \mid w = v_x^a \text{ where } a \in \mathbb{R}\} \\ \llbracket ?\chi \rrbracket &= \perp_{\mathcal{D}} \cup \{(v, \epsilon, v) \mid v \models \chi\} \\ \llbracket \{x' = \theta \ \&x \ \chi\} \rrbracket &= \perp_{\mathcal{D}} \cup \{(\varphi(0), \epsilon, \varphi(s)) \mid \varphi(\zeta) \models \mu' = 1 \wedge x' = \theta \wedge \chi \text{ and} \\ &\quad \varphi(\zeta) = \varphi(0) \text{ on } \{x, \mu\}^{\mathbb{G}} \text{ for all } \zeta \in [0, s] \text{ and a solution} \\ &\quad \varphi : [0, s] \rightarrow \mathcal{S} \text{ with } \varphi(\zeta)(z') = \frac{d\varphi(t)(z)}{dt}(\zeta) \text{ for } z \in \{x, \mu\}\} \end{aligned}$$

⁸ The alternative condition $z \in \mathcal{BV}(\beta)$ leads to an equivalent definition when $w_{\alpha} = w_{\beta}$ on $(\mathcal{BV}(\alpha) \cup \mathcal{BV}(\beta))^{\mathbb{G}}$, which is the case for the final states in parallel composition.

⁹ We consider $w_{\alpha}(\mu) = w_{\beta}(\mu)$ fulfilled if $w_{\alpha} = \perp$ or $w_{\beta} = \perp$.

$$\begin{aligned}
\llbracket ch(h)! \theta \rrbracket &= \{(v, \tau, w) \mid (\tau, w) \preceq (\langle h, ch, \llbracket \theta \rrbracket v, v(\mu) \rangle, v)\} \\
\llbracket ch(h)?x \rrbracket &= \{(v, \tau, w) \mid (\tau, w) \preceq (\langle h, ch, a, v(\mu) \rangle, v_x^a) \text{ where } a \in \mathbb{R}\} \\
\llbracket \alpha \cup \beta \rrbracket &= \llbracket \alpha \rrbracket \cup \llbracket \beta \rrbracket \\
\llbracket \alpha; \beta \rrbracket &= \llbracket \alpha \rrbracket \hat{\circ} \llbracket \beta \rrbracket = \llbracket \alpha \rrbracket_{\perp} \cup (\llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket) \\
\llbracket \alpha^* \rrbracket &= \bigcup_{n \in \mathbb{N}} \llbracket \alpha \rrbracket^n = \bigcup_{n \in \mathbb{N}} \llbracket \alpha^n \rrbracket \\
\llbracket \alpha \parallel \beta \rrbracket &= \llbracket \alpha \rrbracket \parallel \llbracket \beta \rrbracket
\end{aligned}$$

In the semantics of continuous evolution, the solution for the ODE gives meaning to the primed variable x' as in $d\mathcal{L}$ [33]. By $\mu' = 1$, the global time μ always evolves with slope 1 with every continuous evolution.

The semantics $\llbracket \alpha \rrbracket \subseteq \mathcal{D}$ is prefix-closed and total (Def. 4) for every program α (see Appendix A). For atomic non-communicating programs, $\perp_{\mathcal{D}} = \mathcal{S} \times \{\epsilon\} \times \{\perp\}$ ensures prefix-closedness. If $? \chi$ or $\{x' = \theta \& \chi\}$ abort, $\perp_{\mathcal{D}}$ also guarantees totality. Keeping the unfinished computations $\llbracket \alpha \rrbracket_{\perp}$ preserves prefix-closedness of $\alpha; \beta$.

Formula Semantics. The semantics of the first-order fragment is as usual. Like in dynamic logic [11], the box $[\alpha]\psi$ means that ψ is true after all finished computations, i.e., the final state and communication of $(v, \tau, w) \in \llbracket \alpha \rrbracket$ with $w \neq \perp$. The ac-box $[\alpha]_{\{A, C\}}\psi$ additionally means that the communication of (un)finished computations fulfills commitment C. In our modal treatment of ac-reasoning [43], assumption A and program α determine the reachable worlds together, i.e., only computations need to be considered whose incoming communication meets A.

Definition 6 (Formula semantics). *The semantics $\llbracket \varphi \rrbracket \subseteq \mathcal{S}$ of a formula $\varphi \in \text{Fml}(V, \Omega)$ is defined as $\llbracket \varphi \rrbracket = \{v \mid v \models \varphi\}$ using the satisfaction relation \models . The relation \models is defined by induction on the structure of φ as follows:*

1. $v \models e_1 = e_2$ if $\llbracket e_1 \rrbracket v = \llbracket e_2 \rrbracket v$. Accordingly, for $\eta_1 \geq \eta_2$, $ie_1 \geq ie_2$, $te_1 \preceq te_2$
2. $v \models \varphi \wedge \psi$ if $v \models \varphi$ and $v \models \psi$. Accordingly, for \neg, \vee, \rightarrow
3. $v \models \forall z \varphi$ if $v_z^d \models \varphi$ for all $d \in \text{type}(z)$
4. $v \models \exists z \varphi$ if $v_z^d \models \varphi$ for some $d \in \text{type}(z)$
5. $v \models [\alpha]\psi$ if $w \cdot \tau \models \psi$ for all $(v, \tau, w) \in \llbracket \alpha \rrbracket$ with $w \neq \perp$
6. $v \models [\alpha]_{\{A, C\}}\psi$ if for all $(v, \tau, w) \in \llbracket \alpha \rrbracket$ the following conditions hold:

$$\{v \cdot \tau' \mid \tau' \prec \tau\} \models A \text{ implies } v \cdot \tau \models C \quad (\text{commit})$$

$$(\{v \cdot \tau' \mid \tau' \preceq \tau\} \models A \text{ and } w \neq \perp) \text{ implies } w \cdot \tau \models \psi \quad (\text{post})$$

Where $U \models \varphi$ for a set of states $U \subseteq \mathcal{S}$ and any formula $\varphi \in \text{Fml}(V, \Omega)$ if $v \models \varphi$ for all $v \in U$. In particular, $\emptyset \models \varphi$.

In item 6, (commit) is checked after each communication event as desired since *all* communication prefixes are reachable worlds by prefix-closedness of the program semantics $\llbracket \alpha \rrbracket$ (Def. 4). Via state-trace concatenation $v \cdot \tau$ and $w \cdot \tau$ in item 5 and item 6, the communication events recorded in τ become observable. This follows the realization that the reachable worlds of a CHP consist of the final state and the communication trace.

Remark 1. In (commit), assumptions are only available about the communication *strictly* before to prevent unsound circular reasoning [17, 43]. With a non-strict definition, the formula $y = 0 \rightarrow [\text{ch}(h)!y]_{\{A,C\}} \top$, where $A \equiv C \equiv |h \downarrow \text{ch}| > 0 \rightarrow \text{val}(h \downarrow \text{ch}) = 1$, would get valid. Locally, we are aware of the contradiction between the precondition $y = 0$ and what is assumed by A , whereas the environment is not and would trust in the promise C .

Proposition 1 (Conservative extension). *The logic $d\mathcal{L}_{\text{CHP}}$ is a conservative extension of $d\mathcal{L}$. That is, a formula $\varphi \in \text{Fml}(V, \Omega) \cap \text{Fml}_{d\mathcal{L}}$ is valid in $d\mathcal{L}_{\text{CHP}}$ iff it is valid in $d\mathcal{L}$, where $\text{Fml}_{d\mathcal{L}}$ is the set of $d\mathcal{L}$ formulas (see Appendix A).*

2.3 Calculus

This section develops a sound (see Theorem 1) proof calculus for $d\mathcal{L}_{\text{CHP}}$, summarized in Fig. 4 on page 13. In Fig. 5 on page 14, we provide common derived rules. Since $d\mathcal{L}_{\text{CHP}}$ is a conservative extension of $d\mathcal{L}$ (Proposition 1), the entire $d\mathcal{L}$ sequent calculus [33, 34, 36] can be used soundly for reasoning about $d\mathcal{L}_{\text{CHP}}$ formulas. A *sequent* $\Gamma \vdash \Delta$ with finite lists of formulas Γ, Δ is short for $\bigwedge_{\varphi \in \Gamma} \varphi \rightarrow \bigvee_{\psi \in \Delta} \psi$.

Each program statement is axiomatized by a dynamic box $[\cdot]\psi$ or an ac-box $[\cdot]_{\{A,C\}}\psi$. Axioms $[\epsilon]_{\mathcal{AC}}$ and $[\top, \top]$ for switching between dynamic and ac-boxes mediate between them. The dynamic axioms are as usual in differential [33] dynamic logic [11]. The ac-axioms re-express Hoare-style ac-reasoning [43] as a dynamic logic. However, we design more atomic axioms for parallel composition and communication from which the proof rule $[\parallel]_{\mathcal{AC}}\text{R}$ for parallel composition and proof rules for communication derive.

Noninterference (Def. 7) identifies valid instances of the formula $[\alpha]_{\{A,C\}}\psi \rightarrow [\alpha \parallel \beta]_{\{A,C\}}\psi$, which we introduce as axiom $[\parallel]_{\mathcal{AC}}$ in Fig. 4. For formula χ , the accessed channels $CN(\chi) \subseteq \Omega$ are those channels whose communication may influence the truth value of χ , e.g., ch in $|h \downarrow \text{ch}| > 0$. That is, $CN(\chi)$ plays a similar role for the communication traces $v|_{V_{\mathcal{T}}}$ (the state restricted to $V_{\mathcal{T}}$) with $v \in \mathcal{S}$ as $FV(\chi)$ does for the overall state v . For program α , the set $CN(\alpha)$ denotes the communication channels used.

Definition 7 (Noninterference). *Given an ac-box $[\alpha \parallel \beta]_{\{A,C\}}\psi$ the CHP β does not interfere with its surrounding contract if the following conditions hold:¹⁰*

$$FV(\psi) \cap BV(\beta) \subseteq \{\mu\} \cup V_{\mathcal{T}} \quad (1)$$

$$FV(\chi) \cap BV(\beta) \subseteq V_{\mathcal{T}} \quad (\text{for } \chi \in \{A, C\}) \quad (2)$$

$$CN(\chi) \cap CN(\beta) \subseteq CN(\alpha) \quad (\text{for } \chi \in \{A, C, \psi\}) \quad (3)$$

¹⁰ The definition only restricts β 's influence on formulas A, C, ψ but not on program α because in parallel composition $\alpha \parallel \beta$, the subprograms must not share state anyway.

Clearly, state variables bound by β and free in $\chi \in \{\mathbf{A}, \mathbf{C}, \psi\}$ would influence χ 's truth in $[\alpha \parallel \beta]_{\{\mathbf{A}, \mathbf{C}\}}\psi$. But equation (1) and equation (2) do not capture trace variables $V_{\mathcal{T}}$ since they are also α 's interface with communication that might be joint with β . However, equation (3) restricts access to trace variables in χ to those channels whose communication can be observed either exclusively from α or as joint communication between α and β , thus prevents influence of β on χ beyond what is already caused by α . Still Def. 7 allows full access to α 's communication including the joint communication with β .

Dynamic ac-reasoning. In Hoare-style ac-reasoning [43], a distinguished history variable records communication globally. Assuming that h is such a variable in $d\mathcal{L}_{\text{CHP}}$, a tempting but *wrong* axiomatization of the send statement would be

$$[\text{ch}!\theta]\psi(h) \leftrightarrow \forall h_0 (h_0 = h \cdot \langle \text{ch}, \theta, \mu \rangle \rightarrow \psi(h_0)). \quad (\ddagger)$$

Applying it to

$$\vdash [\text{ch}_1!\theta_1][\text{ch}_2!\theta_2]\psi(h) \quad (4)$$

results in $h_0 = h \cdot \langle \text{ch}_1, \theta_1, \mu \rangle \vdash [\text{ch}_2!\theta_2]\psi(h_0)$. After this step, the ongoing history is h_0 . However, another application leads to $h_0 = h \cdot \langle \text{ch}_1, \theta_1, \mu \rangle, h_1 = h \cdot \langle \text{ch}_2, \theta_2, \mu \rangle \vdash \psi(h_0)$. Incorrectly, communication is appended to h again and $\psi(h_0)$ still refers to h_0 . Problematically, the substitution $([\text{ch}_2!\theta_2]\psi(h))\{h \mapsto h_0\}$ during the first application does not guide $\text{ch}_2!\theta_2$ to append its communication to h_0 . Without h occurring syntactically but being free in $\text{ch}_2!\theta_2$ the substitution does not even have a meaningful definition. For a similar reason, axiom

$$[\text{ch}!\theta]\psi(h) \leftrightarrow \psi(h \cdot \langle \text{ch}, \theta, \mu \rangle) \quad (\ddagger)$$

is unsound as applying it twice to equation (4) leads to $\vdash \psi(h \cdot \langle \text{ch}_2, \theta_2, \mu \rangle \cdot \langle \text{ch}_1, \theta_1, \mu \rangle)$ with the communication items in wrong order. Here the axiom is not able to append the second item at the right position because there is no symbolic name for the state $h \cdot \langle \text{ch}_1, \theta_1, \mu \rangle$ of history after the first application.

To enable symbolic execution, we drop the assumption of a distinguished history variable and annotate each communication statement $\text{ch}(h)!\theta$ and $\text{ch}(h)?x$ with an explicit recorder variable h . Now, substitution $\alpha\{h \mapsto h_0\}$ is defined easily as $\text{ch}(h_0)!\theta$ for $\alpha \equiv \text{ch}(h)!\theta$, and as $\text{ch}(h_0)?x$ for $\alpha \equiv \text{ch}(h)?x$, and as α for other atomic programs, and by recursive application otherwise.

Atomic Hybrid Programs. For an (atomic) non-communicating program α , we can flatten $[\alpha]_{\{\mathbf{A}, \mathbf{C}\}}\psi$ by axiom $[\epsilon]_{\mathbf{AC}}$ to a dynamic formula because \mathbf{A} and \mathbf{C} only refer to the initial state when $CN(\alpha) = \emptyset$. Subsequently, we can execute the program by its dynamic axiom ($[\text{:=}], [\text{:*}], [\text{?}], [\text{'}]$). Note that by conservative extension (Proposition 1) axiom $[\text{'}]$ only applies to $[\{x' = \theta \ \& \ \chi\}]\psi$ if the ODE $x' = \theta$ matches the underlying semantics, i.e., if $\mu \in x$, which has right-hand side 1 for well-formed continuous evolution $x' = \theta$. Therefore, axiom $[\mu]$ allows to materialize the flow of global time μ as evolution $\mu' = 1$ whenever necessary.

$$\begin{array}{l}
 [:=] \quad [x := \theta]\psi(x) \leftrightarrow \psi(\theta) \quad [;]_{\mathcal{AC}} \quad [\alpha; \beta]_{\{A,C\}}\psi \leftrightarrow [\alpha]_{\{A,C\}}[\beta]_{\{A,C\}}\psi \\
 [:=*] \quad [x := *]\psi \leftrightarrow \forall x \psi \quad [\cup]_{\mathcal{AC}} \quad [\alpha \cup \beta]_{\{A,C\}}\psi \leftrightarrow [\alpha]_{\{A,C\}}\psi \wedge [\beta]_{\{A,C\}}\psi \\
 [?] \quad [?\chi]\psi \leftrightarrow (\chi \rightarrow \psi) \quad [*]_{\mathcal{AC}} \quad [\alpha^*]_{\{A,C\}}\psi \leftrightarrow [\alpha^0]_{\{A,C\}}\psi \wedge [\alpha]_{\{A,C\}}[\alpha^*]_{\{A,C\}}\psi^a \\
 [\top, \top] \quad [\alpha]\psi \leftrightarrow [\alpha]_{\{\top, \top\}}\psi \quad \mathbf{I}_{\mathcal{AC}} \quad [\alpha^*]_{\{A,C\}}\psi \leftrightarrow [\alpha^0]_{\{A,C\}}\psi \wedge [\alpha^*]_{\{A, \top\}}(\psi \rightarrow [\alpha]_{\{A,C\}}\psi)^a \\
 [\mu] \quad [\{x' = \theta \ \& \ \chi\}]\psi \leftrightarrow [\{\mu' = 1, x' = \theta \ \& \ \chi\}]\psi \\
 ['] \quad [x' = \theta(x)]\psi(x) \leftrightarrow \forall t \geq 0 [x := y(t)]\psi(x) \quad (y'(t) = \theta(y) \text{ and } \mu \in x)^b \\
 [\text{ch}!] \quad [\text{ch}(h)!\theta]\psi(h) \leftrightarrow \forall h_0 (h_0 = h \cdot \langle \text{ch}, \theta, \mu \rangle \rightarrow \psi(h_0)) \quad (h_0 \text{ fresh}) \\
 [\text{ch}!]_{\mathcal{AC}} \quad [\text{ch}(h)!\theta]_{\{A,C\}}\psi \leftrightarrow \mathbf{C} \wedge \left(\mathbf{A} \rightarrow [\text{ch}(h)!\theta](\mathbf{C} \wedge (\mathbf{A} \rightarrow \psi)) \right) \\
 [\text{ch}?]_{\mathcal{AC}} \quad [\text{ch}(h)?x]_{\{A,C\}}\psi \leftrightarrow [x := *][\text{ch}(h)!x]_{\{A,C\}}\psi \\
 [\mathbf{K}] \quad (\text{Cl}_{\forall} \mathbf{K}) \wedge [\alpha]_{\{A_1 \wedge A_2, C_1 \wedge C_2\}}\psi \rightarrow [\alpha]_{\{A, C_1 \wedge C_2\}}\psi^{cd} \\
 [\|_]_{\mathcal{AC}} \quad [\alpha]_{\{A,C\}}\psi \rightarrow [\alpha \|\ \beta]_{\{A,C\}}\psi \quad (\beta \text{ does not interfere with } [\alpha]_{\{A,C\}}\psi \text{ (Def. 7)}) \\
 [\epsilon]_{\mathcal{AC}} \quad [\alpha]_{\{A,C\}}\psi \leftrightarrow \mathbf{C} \wedge (\mathbf{A} \rightarrow [\alpha]\psi) \quad (\text{CN}(\alpha) = \emptyset)^d \quad \mathbf{M}[\cdot]_{\mathcal{AC}} \quad \frac{\mathbf{A}_2 \rightarrow \mathbf{A}_1 \quad \mathbf{C}_1 \rightarrow \mathbf{C}_2 \quad \psi_1 \rightarrow \psi_2}{[\alpha]_{\{A_1, C_1\}}\psi_1 \rightarrow [\alpha]_{\{A_2, C_2\}}\psi_2} \\
 \mathbf{W}[\cdot]_{\mathcal{AC}} \quad [\alpha]_{\{A,C\}}\psi \leftrightarrow \mathbf{C} \wedge [\alpha]_{\{A,C\}}(\mathbf{C} \wedge (\mathbf{A} \rightarrow \psi)) \\
 [\wedge]_{\mathcal{AC}} \quad [\alpha]_{\{A, C_1 \wedge C_2\}}(\psi_1 \wedge \psi_2) \leftrightarrow \bigwedge_{j=1}^2 [\alpha]_{\{A, C_j\}}\psi_j \quad \mathbf{G}_{\mathcal{AC}} \quad \frac{\mathbf{C} \wedge \psi}{[\alpha]_{\{A,C\}}\psi}
 \end{array}$$

^a Note that $[\alpha^0]_{\{A,C\}}\psi \leftrightarrow \mathbf{C} \wedge (\mathbf{A} \rightarrow \psi)$ by $[\epsilon]_{\mathcal{AC}}$ and $[?]$ since $\alpha^0 \equiv ?\top$.

^b Conservative extension only applies if $\mu' = 1$ is part of $x' = \theta$, which holds if $\mu \in x$ since every right-hand side for μ other than 1 in an evolution is considered ill-formed.

^c \mathbf{K} is the compositionality condition $(\mathbf{A} \wedge \mathbf{C}_1 \rightarrow \mathbf{A}_2) \wedge (\mathbf{A} \wedge \mathbf{C}_2 \rightarrow \mathbf{A}_1)$. The universal closure $\text{Cl}_{\forall} \varphi$ of φ is defined by $\forall z_1, \dots, z_n \varphi$, where $FV(\varphi) = \{z_1, \dots, z_n\}$.

^d Care must be taken, for example, when $[\epsilon]_{\mathcal{AC}}$ is applied from right to left, that resulting ac-boxes are well-formed, i.e., $(FV(\mathbf{A}) \cup FV(\mathbf{C})) \cap BV(\alpha) \subseteq V_{\mathcal{T}}$ for $[\alpha]_{\{A,C\}}\psi$.

Fig. 4: $d\mathcal{L}_{\text{CHP}}$ proof calculus

Compound Hybrid Programs. Compound CHPs in an ac-box cannot be handled by axiom $[\epsilon]_{\mathcal{AC}}$ if they communicate. Instead, they have the axioms $[;]_{\mathcal{AC}}$, $[\cup]_{\mathcal{AC}}$, $[*]_{\mathcal{AC}}$, and $\mathbf{I}_{\mathcal{AC}}$. Compound programs in a dynamic box can be repackaged into an ac-box using axiom $[\top, \top]$. The ac-induction axiom $\mathbf{I}_{\mathcal{AC}}$ carefully generalizes that of dynamic logic [11]. Importantly, the reachable worlds, where the induction step needs to hold, only depend on program α and assumption \mathbf{A} about incoming communication, whereas commitment \mathbf{C} is proven inductively. In Fig. 5, we give the useful derived loop invariant proof rule $\text{ind}_{\mathcal{AC}}$. As usual, it derives from axiom $\mathbf{I}_{\mathcal{AC}}$ and the ac-version $\mathbf{G}_{\mathcal{AC}}$ of the Gödel-generalization rule, which confirms that the embedding into dynamic logic is correct.

If we were to neglect the communication of aborting runs in the semantics of $\alpha; \beta$, axiom $[;]_{\mathcal{AC}}$ would not be sound. Proving $[\alpha]_{\{A,C\}}[\beta]_{\{A,C\}}\psi$ requires to show the commitment \mathbf{C} after each communication of α even if β aborts. To obtain this from $[\alpha; \beta]_{\{A,C\}}\psi$ the semantics $\llbracket \alpha; \beta \rrbracket = \llbracket \alpha \rrbracket_{\perp} \cup (\llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket)$ contains all respective runs of α up to β with $\llbracket \alpha \rrbracket_{\perp}$.

$$\begin{array}{c}
\text{CG} \frac{\Gamma, h_0 = h \cdot \langle \text{ch}, \theta, \mu \rangle \vdash [\alpha(h_0)]\psi, \Delta}{\Gamma, \vdash [\alpha(h)]\psi, \Delta} \quad (h \notin \text{FV}(\psi) \text{ and } h_0 \text{ fresh}) \\
\text{ind}_{\mathcal{AC}} \frac{\Gamma \vdash C \wedge I, \Delta \quad C, I \vdash [\alpha]_{\{A, C\}} I \quad A, C, I \vdash \psi}{\Gamma \vdash [\alpha^*]_{\{A, C\}} \psi, \Delta} \quad \text{[if]} \frac{\Gamma, \varphi \vdash [\alpha]\psi, \Delta \quad \Gamma, \neg\varphi \vdash \psi, \Delta}{\Gamma \vdash \text{[if}(\varphi)\{\alpha\}\psi, \Delta} \\
\text{[||-]}_{\mathcal{ACR}} \frac{\vdash K \quad \Gamma \vdash [\alpha_j]_{\{A_j, C_j\}} \psi_j, \Delta \quad (j = 1, 2)}{\Gamma \vdash [\alpha_1 \parallel \alpha_2]_{\{A, C_1 \wedge C_2\}} (\psi_1 \wedge \psi_2), \Delta} \quad (\alpha_{3-j} \text{ does not interfere with } [\alpha_j]_{\{A_j, C_j\}} \psi_j \text{ for } j = 1, 2 \text{ (Def. 7)})^a \\
\text{[!}\theta\text{]}_{\mathcal{ACR}} \frac{\Gamma \vdash C(h), \Delta \quad \Gamma, H_0 \vdash C(h_0), \Delta \quad \Gamma, H_0, A(h_0) \vdash \psi(h_0), \Delta}{\Gamma \vdash \text{[ch}(h)\text{!}\theta]_{\{A(h), C(h)\}} \psi(h), \Delta} \quad (h_0 \text{ fresh})^b
\end{array}$$

^a K is the compositionality condition $(A \wedge C_1 \rightarrow A_2) \wedge (A \wedge C_2 \rightarrow A_1)$.

^b Formula H_0 is short for $h_0 = h \cdot \langle \text{ch}, \theta, \mu \rangle$ recording the global time μ as timestamp.

Fig. 5: Derived $d\mathcal{L}_{\text{CHP}}$ proof rules

Communication Statements. Axiom $\text{[ch!]}_{\mathcal{AC}}$ unfolds (commit) for an ac-box of a single send statement into a dynamic box. The effect on the recorder variable h of executing sending $\text{ch}(h)\text{!}\theta$ is captured by axiom [ch!] . It records the event $\langle \text{ch}, \theta, \mu \rangle$ using the current global time μ as timestamp and renames the history in the postcondition for subsequent proof steps. Axiom $\text{[ch?]}_{\mathcal{AC}}$ allows to execute a receive statement by its duality with send. Derived rule $\text{[!}\theta\text{]}_{\mathcal{ACR}}$ combines $\text{[}\epsilon\text{]}_{\mathcal{AC}}$ and [ch!] , and decomposes the statement into two premises for (commit) and one for (post). Derived rule CG is useful as it eliminates the need for case distinction about empty history by prefixing the history with additional ghost communication.

Parallel Composition. A non-interfering program β (Def. 7) can be dropped from parallel composition $[\alpha \parallel \beta]_{\{A, C\}} \psi$ by axiom $\text{[||-]}_{\mathcal{AC}}$ because it has no influence on the surrounding contract $[\alpha \parallel _]_{\{A, C\}} \psi$. Since \parallel is associative and commutative (see Appendix C), axiom $\text{[||-]}_{\mathcal{AC}}$ can drop any subprogram in a chain of parallel statements. In parallel composition of $[\alpha_j]_{\{A_j, C_j\}} \psi_j$ for $j = 1, 2$, the commitments mutually contribute to the assumptions. This can weaken the assumption of $\alpha_1 \parallel \alpha_2$ about its environment to A by axiom [K] if the compositionality condition $K \equiv (A \wedge C_1 \rightarrow A_2) \wedge (A \wedge C_2 \rightarrow A_1)$ is valid.

The derived rule $\text{[||-]}_{\mathcal{ACR}}$ combines axioms $\text{[||-]}_{\mathcal{AC}}$ and [K] for full decomposition of parallelism. Reasoning about a parallel $[\alpha_1 \parallel \alpha_2]_{\{A, C\}} \psi$ with arbitrary A, C , and ψ is the task of constructing A_j, C_j , and ψ_j for $j = 1, 2$ such that $C_1 \wedge C_2 \rightarrow C$, and $\psi_1 \wedge \psi_2 \rightarrow \psi$ are valid, and such that $\text{[||-]}_{\mathcal{ACR}}$ is applicable. Since the side condition of $\text{[||-]}_{\mathcal{ACR}}$ about noninterference still allows C_j and ψ_j for $j = 1, 2$ to access α_j 's communication including the joint communication with α_{3-j} , the formulas can cover the complete communication of $\alpha_1 \parallel \alpha_2$.

Miscellaneous. Ac-boxes distribute over conjunctions by axiom $\text{[}\wedge\text{]}_{\mathcal{AC}}$ except for assumptions just like preconditions φ_j do not distribute in $\varphi_1 \wedge \varphi_2 \rightarrow [\alpha]\psi$.

Rule $M[\cdot]_{\mathcal{AC}}$ generalizes monotonicity from dynamic to ac-boxes. Ac-weakening $W[\cdot]_{\mathcal{AC}}$ exploits totality of the program semantics $\llbracket \alpha \rrbracket$ to add or drop C in the initial state. Moreover, adding or dropping C and $A \rightarrow \psi$ by $W[\cdot]_{\mathcal{AC}}$ in the final state is due to (commit) and (post), respectively. Rule $G_{\mathcal{AC}}$ is the ac-version of the Gödel-generalization rule.

First-order formulas $FOL_{\mathbb{N}}(V_{\mathbb{N}})$ over $\text{Trm}_{\mathbb{N}}(V, \Omega)$ without length computations $|te|$, can be handled by an effective oracle proof rule (called \mathbb{N}) since Presburger arithmetic is decidable [37]. Likewise, first-order real arithmetic $FOL_{\mathbb{R}}(V_{\mathbb{R}})$ is decidable [40], and we use an oracle rule for it (called \mathbb{R}) as in $d\mathcal{L}$ [30]. However, the full first-order fragment $FOL(V, \Omega)$ of $d\mathcal{L}_{\text{CHP}}$ is not decidable because of alternating quantifiers of trace and integer variables [5].

Instead, reasoning about trace terms is by simple algebraic laws for successive simplification (see Appendix C) [42]. For applicability of rules \mathbb{R} and \mathbb{N} trace subterms can be rewritten with fresh variables. For example, $\text{val}(te_1[ie]) < \text{val}(te_2[ie]) \rightarrow \text{val}(te_1[ie]) + \theta < \text{val}(te_2[ie]) + \theta$ is valid since $x < y \rightarrow x + \theta < y + \theta$ is valid in $FOL_{\mathbb{R}}(V_{\mathbb{R}})$. Ultimately, we use \mathbb{N} and \mathbb{R} modulo trace terms, i.e., perform rewritings silently.

Our central contribution is the soundness theorem about the compositional $d\mathcal{L}_{\text{CHP}}$ proof calculus. The proof rules given in Fig. 5 derive (see Appendix C).

Theorem 1. *The $d\mathcal{L}_{\text{CHP}}$ calculus (see Fig. 4) is sound (see Appendix C).*

3 Demonstration of $d\mathcal{L}_{\text{CHP}}$

We demonstrate our calculus outlining a proof of the safety condition from Example 3 on page 7 about the convoy in Fig. 2 on page 6. After decomposing the parallel statement, the proof proceeds purely mechanical by statement-by-statement symbolic execution. It starts in Fig. 7 using a standard pattern for decomposing a parallel statement: First, introduce commitments and postconditions (see Fig. 6) by axiom $\llbracket \tau, \tau \rrbracket$ and rule $M[\cdot]_{\mathcal{AC}}$, which relate the subprograms, such that second, rule $\llbracket \llbracket \cdot \rrbracket \rrbracket_{\mathcal{AC}} \mathbb{R}$ becomes applicable. The latter also makes the commitments mutual assumptions.

The formulas in Fig. 6 relating the subprograms clearly reduce their complex interior and only depend on the small communication interface and local variables, thus are independent of any knowledge about the internal structure of the respective other car. Adding initial ghost communication (rule CG) exploits the flexibility of explicit history variables and dynamic logic to avoid cumbersome case distinction about empty history.

Fig. 7 decomposes $x_f < x_l$ into ψ_f and ψ_l since **follower** stays behind **leader**'s last known position $\text{val}(h_{\text{pos}})$, whereas **leader** never drives backward by ψ_l . Indeed, **follower** stores the last known distance $\text{val}(h_{\text{pos}}) - x_f$ in d and $\epsilon - \Delta(\mu, h_{\text{pos}})$ bounds the waiting time till the next position update along channel pos. Thus, **follower** stays behind **leader** when driving with speed d/ϵ .

Fig. 8 continues from Fig. 7 and demonstrates the reasoning along one execution path of **follower**. The invariant F for induction $\text{ind}_{\mathcal{AC}}$ bounds **follower**'s

$\frac{*}{F, A, 0 \leq v_{tar} \leq V, d > \epsilon V, t \geq 0 \vdash 0 \leq v_{tar} \leq d/\epsilon \wedge v_f \leq V \wedge x_f + (\epsilon - \Delta(\mu + t, h \downarrow \text{pos}))^{d/\epsilon} < \mathbf{val}(h \downarrow \text{pos})}$	\mathbb{R}
$F, A, A(h \cdot \langle \text{ch}, v_{tar}, \mu \rangle), d > \epsilon V, t \geq 0 \vdash F(v_{tar}, x_f + t \cdot v_{tar}, \mu + t, h_{\text{vel}})$	TA
$F, A, H_{\text{vel}}, A(h_{\text{vel}}), d > \epsilon V, t \geq 0 \vdash F(v_{tar}, x_f + t \cdot v_{tar}, \mu + t, h_{\text{vel}})$	$=\text{L}$
$F, A, H_{\text{vel}}, A(h_{\text{vel}}), d > \epsilon V \vdash \forall t \geq 0 F(v_{tar}, x_f + t \cdot v_{tar}, \mu + t, h_{\text{vel}})$	$\forall \mathbb{R}$
$F, A, H_{\text{vel}}, A(h_{\text{vel}}), d > \epsilon V \vdash [\mu' = 1, x'_f = v_{tar}]F(v_{tar}, x_f, \mu, h_{\text{vel}})$	$[\cdot], [:=]$
$F, A, H_{\text{vel}}, A(h_{\text{vel}}), d > \epsilon V \vdash [\mathbf{plant}_f(v_{tar})]F(v_{tar}, x_f, \mu, h_{\text{vel}})$	$[\mu]$
$F, A, H_{\text{vel}}, A(h_{\text{vel}}), d > \epsilon V \vdash [v_f := v_{tar}][\mathbf{plant}_f(v_f)]F(v_f, x_f, \mu, h_{\text{vel}}) \triangleright \text{Fig. 26}$	$[:=]$
$F, A, H_{\text{vel}}, A(h_{\text{vel}}) \vdash [\mathbf{if}(\text{safe}_d) v_f := v_{tar}][\mathbf{plant}_f]F(v_f, x_f, \mu, h_{\text{vel}}) \triangleright \text{TR}$	$[\mathbf{if}]$
$F \vdash [\mathbf{vel}(h)!v_{tar}]_{\{A(h), \tau\}}[\mathbf{if}(\text{safe}_d) v_f := v_{tar}][\mathbf{plant}_f]F(v_f, x_f, \mu, h) \triangleright \text{TR}$	$[\theta]_{\mathcal{ACR}}$
$F \vdash \forall v_{tar} [\mathbf{vel}(h)!v_{tar}]_{\{A(h), \tau\}}[\mathbf{if}(\text{safe}_d) v_f := v_{tar}][\mathbf{plant}_f]F(v_f, x_f, \mu, h)$	$\forall \mathbb{R}$
$F \vdash [v_{tar} := *][\mathbf{vel}(h)!v_{tar}]_{\{A(h), \tau\}}[\mathbf{if}(\text{safe}_d) v_f := v_{tar}][\mathbf{plant}_f]F(v_f, x_f, \mu, h)$	$[:=]$
$F \vdash [\mathbf{vel}(h)?v_{tar}]_{\{A(h), \tau\}}[\mathbf{if}(\text{safe}_d) v_f := v_{tar}][\mathbf{plant}_f]F(v_f, x_f, \mu, h)$	$[\text{ch?}]_{\mathcal{AC}}$
$F \vdash [\mathbf{vel}(h)?v_{tar}]_{\{A(h), \tau\}}[\mathbf{if}(\text{safe}_d) v_f := v_{tar}]_{\{A(h), \tau\}}[\mathbf{plant}_f]F(v_f, x_f, \mu, h)$	$[\epsilon]_{\mathcal{AC}}, W[\]_{\mathcal{AC}}$
$F \vdash [\mathbf{velo}(h)]_{\{A(h), \tau\}}[\mathbf{plant}_f]F(v_f, x_f, \mu, h) \triangleright \text{Fig. 27}$	$[:]_{\mathcal{AC}}$
Execution by $[\cdot]_{\mathcal{AC}}, [\cup]_{\mathcal{AC}}, [\epsilon]_{\mathcal{AC}}, W[\]_{\mathcal{AC}}$, and $\wedge \mathbb{R}$	
$F \vdash [(\mathbf{velo}(h) \cup \mathbf{dist}(h)); \mathbf{plant}_f]_{\{A(h), \tau\}}F(v_f, x_f, \mu, h) \triangleright_1 \text{TA}, \mathbb{R} \triangleright_2 \mathbb{R}$	$\text{ind}_{\mathcal{AC}}$
$\Gamma \vdash [\mathbf{follower}(h)]_{\{A(h), \tau\}}\psi_f$	

Fig. 8: Partial proof for `follower`. The induction $\text{ind}_{\mathcal{AC}}$ uses $F \equiv 0 \leq v_f \leq d/\epsilon \wedge v_f \leq V \wedge x_f + (\epsilon - \Delta(\mu, h \downarrow \text{pos}))^{d/\epsilon} < \mathbf{val}(h \downarrow \text{pos})$ as invariant. Formula H_{vel} is short for $h_{\text{vel}} = h \cdot \langle \text{vel}, v_{tar}, \mu \rangle$ and safe_d is short for $d > \epsilon V$. The induction base $\Gamma \vdash F$ (\triangleright_1) closes by trace algebra TA and real arithmetic \mathbb{R} . Postcondition $F \vdash \psi_f$ (\triangleright_2) holds by \mathbb{R} . For clarity, we highlight substitutions.

components [4, 7, 13, 19, 22, 23, 26] would need to be rethought to integrate lossy communication as for every other new application as well.

Hybrid Hoare-logic (HHL) for HCSP [21] is non-compositional [41]. Wang *et al.* [41] extend it with assume-guarantee reasoning (AGR) in a way that, unlike $d\mathcal{L}_{\text{CHP}}$, becomes non-compositional again. Unfortunately, their rule for parallel composition still explicitly unrolls all interleavings in the postcondition for communication traces reflecting the structure of the subprograms. Assumptions and guarantees in HHL cannot specify the communication history but consider readiness for reasoning about deadlock freedom for future work [41]. Externalizing the complete observable behavior (and program structure) in this way devalues the whole point of compositionality [38, Section 1.6.2] but only postpones reasoning about the exponentially many interleavings. Similarly, Guelev *et al.* encode the semantics of the parallel composition into the postcondition [10].

Hoare-style ac-reasoning [16, 43, 44] including Hoare-style reasoning for HCSP [10, 21, 41] lacks symbolic execution as intuitive reasoning principle but manages

with a distinguished history variable since multiple Hoare-triples cannot be considered together. $d\mathcal{L}_{\text{CHP}}$ makes symbolic execution possible despite communication through explicit trace variables referring to the different possible states of the history in a proof. The resulting combination of ac-reasoning and dynamic logic allows flexible switch between first-order, dynamic, and ac-reasoning while the axioms are simple capturing discrete, continuous, or communication behavior. Unlike $d\mathcal{L}_{\text{CHP}}$, which has a global flow of time due to continuous evolution, calculi for distributed real-time computation [16, 17] need to consider the waiting for termination of time-consuming discrete statements.

Unlike other $d\mathcal{L}$ approaches [19, 22, 26], $d\mathcal{L}_{\text{CHP}}$ has a parallel operator with built-in time-synchronization as first-class citizen in hybrid programs that can be arbitrarily nested with other programs, rather than parallel composition of meta-level components with an explicit time model. Modeling of parallelism by nondeterministic choice additionally requires extra care to ensure execution periodicity [22]. In contrast to first-order constraints relating at most consecutive I/O events [19, 22, 26], $d\mathcal{L}_{\text{CHP}}$ can reason about invariants of the whole communication history. Different from our integrated reasoning about discrete, hybrid, and communication behavior, Kamburjan *et al.* [19] separate reasoning about communication from hybrid systems reasoning.

Quantified differential dynamic logic $Qd\mathcal{L}$ [32] allows reasoning about parallel compositions of an unbounded number of distributed CPSs. Unlike $d\mathcal{L}_{\text{CHP}}$ that can reason about interactions of entirely different programs, parallelism in $Qd\mathcal{L}$ is restricted to subprograms with a *homogeneous* structure.

Different from the denotational semantics of CHPs, parallel composition of hybrid automata [4, 8, 13, 23], just like Hoare-style reasoning about HCSP [10, 41], always fall back to the combinatorial exploration of parallelism. Consequently, even AGR approaches [4, 8, 12, 23] for hybrid automata that mitigate the state space explosion for subautomata, eventually resort to large product automata later. In contrast, $d\mathcal{L}_{\text{CHP}}$'s proof rule for parallel composition exploits the built-in compositionality of its semantics enabling verification of subprograms truly independent of their environment except for the communication interface. Unlike ac-formulas in $d\mathcal{L}_{\text{CHP}}$, which can capture change, rate, delay, or noise for arbitrary pairings of communication channels, overapproximation is limited to coarse abstractions by timed transition systems [8], components completely dropping knowledge about continuous behavior [13], or static global contracts [4]. Where $d\mathcal{L}_{\text{CHP}}$ inherits complete reasoning about differential equation invariants from $d\mathcal{L}$, automata approaches are often limited to linear continuous dynamics [8, 13].

Concurrent dynamic logic (CDL) has no way for parallel programs to interact [29]. CDL with communication [28] has CSP-style [14] communication primitives but lacks continuous behavior and a proof calculus for verification.

5 Conclusion

This paper presented a dynamic logic $d\mathcal{L}_{\text{CHP}}$ for communicating hybrid programs (CHPs) with synchronous parallel composition in global time. The $d\mathcal{L}_{\text{CHP}}$ proof

calculus is the first truly compositional verification approach for communicating parallel hybrid systems. To this end, $d\mathcal{L}_{\text{CHP}}$ exploits the flexibility of dynamic logic by complementing necessity and possibility modalities with assumption-commitment (ac) modalities. Crucially, this embedding of ac-reasoning enables compositional specification and verification of parallel hybrid behavior in a way that tames their complexity. The practical feasibility of $d\mathcal{L}_{\text{CHP}}$ increases as it supports reasoning via intuitive symbolic execution in the presence of communication. All technical subtleties in the semantic construction remain under the hood such that the actual calculus naturally generalizes dynamic logic reasoning.

Future work includes developing a uniform substitution calculus [33] for $d\mathcal{L}_{\text{CHP}}$ in order to enable parsimonious theorem prover implementations [9].

Funding Statement. This project was funded in part by the Deutsche Forschungsgemeinschaft (DFG) – 378803395 (ConVeY) and an Alexander von Humboldt Professorship.

References

- [1] Alur, R.: Formal verification of hybrid systems. In: Chakraborty, S., Jerraya, A., Baruah, S.K., Fischmeister, S. (eds.) Proc. 11th Intl. Conf. Embedded Software (EMSOFT). pp. 273–278. ACM Press (2011). doi: [10.1145/2038642.2038685](https://doi.org/10.1145/2038642.2038685)
- [2] Alur, R., Courcoubetis, C., Henzinger, T.A., Ho, P.: Hybrid automata: An algorithmic approach to the specification and verification of hybrid systems. In: Grossman, R.L., Nerode, A., Ravn, A.P., Rischel, H. (eds.) Proc. 1th and 2nd Intl. Workshop Hybrid Systems (HS). LNCS, vol. 736, pp. 209–229. Springer (1993). doi: [10.1007/3-540-57318-6_30](https://doi.org/10.1007/3-540-57318-6_30)
- [3] Apt, K.R., de Boer, F.S., Olderog, E.R.: Verification of Sequential and Concurrent Programs. Springer, 3rd edn. (2010). doi: [10.1007/978-1-84882-745-5](https://doi.org/10.1007/978-1-84882-745-5)
- [4] Benvenuti, L., Bresolin, D., Collins, P., Ferrari, A., Geretti, L., Villa, T.: Assume-guarantee verification of nonlinear hybrid systems with ARIADNE. Intl. J. Robust Nonlinear Control **24**, 699–724 (2014). doi: [10.1002/rnc.2914](https://doi.org/10.1002/rnc.2914)
- [5] Bradley, A.R., Manna, Z., Sipma, H.B.: What’s decidable about arrays? In: Emerson, E.A., Namjoshi, K.S. (eds.) Verification, Model Checking, and Abstract Interpretation, 7th International Conference, VMCAI 2006, Charleston, SC, USA, January 8-10, 2006, Proceedings. LNCS, vol. 3855, pp. 427–442. Springer (2006). doi: [10.1007/11609773_28](https://doi.org/10.1007/11609773_28)
- [6] Cong, X., Yu, H., Xu, X.: Verification of hybrid chi model for cyber-physical systems using phaver. In: Barolli, L., You, I., Xhafa, F., Leu, F., Chen, H. (eds.) Seventh International Conference on Innovative Mobile and Internet Services in Ubiquitous Computing, IMIS 2013, Taichung, Taiwan, July 3-5, 2013. pp. 122–128. IEEE Computer Society (2013). doi: [10.1109/IMIS.2013.29](https://doi.org/10.1109/IMIS.2013.29)
- [7] Frehse, G.: Phaver: algorithmic verification of hybrid systems past hytech. Int. J. Softw. Technol. Transf. **10**(3), 263–279 (2008). doi: [10.1007/s10009-007-0062-x](https://doi.org/10.1007/s10009-007-0062-x)
- [8] Frehse, G., Han, Z., Krogh, B.H.: Assume-guarantee reasoning for hybrid I/O-automata by over-approximation of continuous interaction. In: Proc. 43rd IEEE Conf. Decision and Control (CDC). pp. 479–484. IEEE (2004). doi: [10.1109/CDC.2004.1428676](https://doi.org/10.1109/CDC.2004.1428676)
- [9] Fulton, N., Mitsch, S., Quesel, J.D., Völpl, M., Platzer, A.: KeYmaera X: An axiomatic tactical theorem prover for hybrid systems. In: Felty, A., Middeldorp, A. (eds.) CADE. LNCS, vol. 9195, pp. 527–538. Springer, Berlin (2015). doi: [10.1007/978-3-319-21401-6_36](https://doi.org/10.1007/978-3-319-21401-6_36)

- [10] Guelev, D.P., Wang, S., Zhan, N.: Compositional Hoare-style reasoning about hybrid CSP in the duration calculus. In: Larsen, K.G., Sokolsky, O., Wang, J. (eds.) *Dependable Software Engineering. Theories, Tools, and Applications - Third International Symposium, SETTA 2017, Changsha, China, October 23-25, 2017, Proceedings*. LNCS, vol. 10606, pp. 110–127. Springer (2017). doi: [10.1007/978-3-319-69483-2_7](https://doi.org/10.1007/978-3-319-69483-2_7)
- [11] Harel, D.: *First-Order Dynamic Logic*, LNCS, vol. 68. Springer (1979). doi: [10.1007/3-540-09237-4](https://doi.org/10.1007/3-540-09237-4)
- [12] Henzinger, T.A.: The theory of hybrid automata. In: *Proc. 11th IEEE Symp. Logic in Computer Science (LICS)*. pp. 278–292. IEEE (1996). doi: [10.1109/LICS.1996.561342](https://doi.org/10.1109/LICS.1996.561342)
- [13] Henzinger, T.A., Minea, M., Prabhu, V.S.: Assume-guarantee reasoning for hierarchical hybrid systems. In: Benedetto, M.D.D., Sangiovanni-Vincentelli, A.L. (eds.) *Proc. 4th Intl. Workshop Hybrid Systems: Computation and Control (HSCC)*. LNCS, vol. 2034, pp. 275–290. Springer (2001). doi: [10.1007/3-540-45351-2_24](https://doi.org/10.1007/3-540-45351-2_24)
- [14] Hoare, C.A.R.: Communicating sequential processes. *Communications of the ACM* **21**(8), 666–677 (1978). doi: [10.1145/359576.359585](https://doi.org/10.1145/359576.359585)
- [15] Hoare, C.A.R.: *Communicating Sequential Processes*. Prentice Hall International (1985)
- [16] Hooman, J.: A compositional proof theory for real-time distributed message passing. In: de Bakker, J.W., Nijman, A.J., Treleaven, P.C. (eds.) *PARLE, Parallel Architectures and Languages Europe, Volume II: Parallel Languages*, Eindhoven, The Netherlands, June 15-19, 1987, *Proceedings*. LNCS, vol. 259, pp. 315–332. Springer (1987). doi: [10.1007/3-540-17945-3_18](https://doi.org/10.1007/3-540-17945-3_18)
- [17] Hooman, J.J.M., de Roever, W.P.: An introduction to compositional methods for concurrency and their application to real-time. *Sādhanā* **17**(1), 29–73 (1992). doi: [10.1007/BF02811338](https://doi.org/10.1007/BF02811338)
- [18] Jifeng, H.: A classical mind: essays in honour of C. A. R. Hoare, chap. *From CSP to Hybrid Systems*, pp. 171–189. Prentice Hall International (1994)
- [19] Kamburjan, E., Schlatte, R., Johnsen, E.B., Tarifa, S.L.T.: Designing distributed control with hybrid active objects. In: Margaria, T., Steffen, B. (eds.) *Proc. 9th Intl. Symp. Leveraging Applications of Formal Methods : Tools and Trends (ISoLA)*. LNCS, vol. 12479, pp. 88–108. Springer (2020). doi: [10.1007/978-3-030-83723-5_7](https://doi.org/10.1007/978-3-030-83723-5_7)
- [20] Levin, G., Gries, D.: A proof technique for communicating sequential processes. *Acta Informatica* **15**(3), 281–302 (1981). doi: [10.1007/BF00289266](https://doi.org/10.1007/BF00289266)
- [21] Liu, J., Lv, J., Quan, Z., Zhan, N., Zhao, H., Zhou, C., Zou, L.: A calculus for hybrid CSP. In: Ueda, K. (ed.) *Proc. 8th Asian Symp. Programming Languages and Systems (APLAS)*. LNCS, vol. 6461, pp. 1–15. Springer (2010). doi: [10.1007/978-3-642-17164-2_1](https://doi.org/10.1007/978-3-642-17164-2_1)
- [22] Lunel, S., Mitsch, S., Boyer, B., Talpin, J.: Parallel composition and modular verification of computer controlled systems in differential dynamic logic. In: ter Beek, M.H., McIver, A., Oliveira, J.N. (eds.) *Proc. 3rd World Congr. Formal Methods - The Next 30 Years (FM)*. LNCS, vol. 11800, pp. 354–370. Springer (2019). doi: [10.1007/978-3-030-30942-8_22](https://doi.org/10.1007/978-3-030-30942-8_22)
- [23] Lynch, N.A., Segala, R., Vaandrager, F.W.: Hybrid I/O automata. *Information and Computation* **185**(1), 105–157 (2003). doi: [10.1016/S0890-5401\(03\)00067-1](https://doi.org/10.1016/S0890-5401(03)00067-1)
- [24] Man, K.L., Reniers, M.A., Cuijpers, P.J.L.: Case studies in the hybrid process algebra HyPA. *Int. J. Softw. Eng. Knowl. Eng.* **15**(2), 299–306 (2005). doi: [10.1142/S0218194005002385](https://doi.org/10.1142/S0218194005002385)
- [25] Misra, J., Chandy, K.M.: Proofs of networks of processes. *IEEE Transactions on Software Engineering* **7**(4), 417–426 (1981). doi: [10.1109/TSE.1981.230844](https://doi.org/10.1109/TSE.1981.230844)
- [26] Müller, A., Mitsch, S., Retschitzegger, W., Schwinger, W., Platzer, A.: Tactical contract composition for hybrid system component verification. *STTT* **20**(6), 615–643 (2018). doi: [10.1007/s10009-018-0502-9](https://doi.org/10.1007/s10009-018-0502-9), special issue for selected papers from FASE'17
- [27] Owicki, S.S., Gries, D.: An axiomatic proof technique for parallel programs I.

- Acta Informatica **6**, 319–340 (1976). doi: [10.1007/BF00268134](https://doi.org/10.1007/BF00268134)
- [28] Peleg, D.: Communication in concurrent dynamic logic. *J. Comput. Syst. Sci.* **35**(1), 23–58 (1987). doi: [10.1016/0022-0000\(87\)90035-3](https://doi.org/10.1016/0022-0000(87)90035-3)
- [29] Peleg, D.: Concurrent dynamic logic. *J. ACM* **34**(2), 450–479 (1987). doi: [10.1145/23005.23008](https://doi.org/10.1145/23005.23008)
- [30] Platzer, A.: Differential dynamic logic for hybrid systems. *J. Autom. Reas.* **41**(2), 143–189 (2008). doi: [10.1007/s10817-008-9103-8](https://doi.org/10.1007/s10817-008-9103-8)
- [31] Platzer, A.: *Logical Analysis of Hybrid Systems: Proving Theorems for Complex Dynamics*. Springer, Heidelberg (2010). doi: [10.1007/978-3-642-14509-4](https://doi.org/10.1007/978-3-642-14509-4)
- [32] Platzer, A.: Quantified differential dynamic logic for distributed hybrid systems. In: Dawar, A., Veith, H. (eds.) *CSL. LNCS*, vol. 6247, pp. 469–483. Springer (2010). doi: [10.1007/978-3-642-15205-4_36](https://doi.org/10.1007/978-3-642-15205-4_36)
- [33] Platzer, A.: A complete uniform substitution calculus for differential dynamic logic. *J. Autom. Reas.* **59**(2), 219–265 (2017). doi: [10.1007/s10817-016-9385-1](https://doi.org/10.1007/s10817-016-9385-1)
- [34] Platzer, A.: *Logical Foundations of Cyber-Physical Systems*. Springer, Cham (2018). doi: [10.1007/978-3-319-63588-0](https://doi.org/10.1007/978-3-319-63588-0)
- [35] Platzer, A., Tan, Y.K.: Differential equation axiomatization: The impressive power of differential ghosts. In: Dawar, A., Grädel, E. (eds.) *LICS*. pp. 819–828. ACM, New York (2018). doi: [10.1145/3209108.3209147](https://doi.org/10.1145/3209108.3209147)
- [36] Platzer, A., Tan, Y.K.: Differential equation invariance axiomatization. *J. ACM* **67**(1), 6:1–6:66 (2020). doi: [10.1145/3380825](https://doi.org/10.1145/3380825)
- [37] Presburger, M.: Über die Vollständigkeit eines gewissen Systems der Arithmetik ganzer Zahlen, in welchem die Addition als einzige Operation hervortritt. In: *Comptes Rendus du I congrès de Mathématiciens des Pays Slaves* (1931)
- [38] de Roever, W.P., de Boer, F.S., Hannemann, U., Hooman, J.J.M., Lakhnech, Y., Poel, M., Zwiers, J.: *Concurrency Verification: Introduction to Compositional and Noncompositional Methods*, Cambridge Tracts in Theoretical Computer Science, vol. 54. Cambridge University Press (2001)
- [39] Song, H., Compton, K.J., Rounds, W.C.: SPHIN: A model checker for reconfigurable hybrid systems based on SPIN. In: Lazic, R., Nagarajan, R. (eds.) *Proceedings of the 5th International Workshop on Automated Verification of Critical Systems, AVoCS 2005*, University of Warwick, UK, September 12–13, 2005. *ENTCS*, vol. 145, pp. 167–183. Elsevier (2005). doi: [10.1016/j.entcs.2005.10.011](https://doi.org/10.1016/j.entcs.2005.10.011)
- [40] Tarski, A.: *A Decision Method for Elementary Algebra and Geometry*. University of California Press, Berkeley, 2nd edn. (1951). doi: [10.1525/9780520348097](https://doi.org/10.1525/9780520348097)
- [41] Wang, S., Zhan, N., Guelev, D.P.: An assume/guarantee based compositional calculus for hybrid CSP. In: Agrawal, M., Cooper, S.B., Li, A. (eds.) *Proc. 9th Conf. Theory and Applications of Models of Computation (TAMC)*. LNCS, vol. 7287, pp. 72–83. Springer (2012). doi: [10.1007/978-3-642-29952-0_13](https://doi.org/10.1007/978-3-642-29952-0_13)
- [42] Zwiers, J.: *Compositionality, Concurrency and Partial Correctness - Proof Theories for Networks of Processes, and Their Relationship*, LNCS, vol. 321. Springer (1989). doi: [10.1007/BFb0020836](https://doi.org/10.1007/BFb0020836)
- [43] Zwiers, J., de Bruin, A., de Roever, W.P.: A proof system for partial correctness of dynamic networks of processes (extended abstract). In: Clarke, E.M., Kozen, D. (eds.) *Proc. Carnegie Mellon Workshop Logics of Programs 1983*. LNCS, vol. 164, pp. 513–527. Springer (1983). doi: [10.1007/3-540-12896-4_384](https://doi.org/10.1007/3-540-12896-4_384)
- [44] Zwiers, J., de Roever, W.P., van Emde Boas, P.: Compositionality and concurrent networks: Soundness and completeness of a proofsystem. In: Brauer, W. (ed.) *Proc. 12th Intl. Coll. Automata, Languages and Programming (ICALP)*. LNCS, vol. 194, pp. 509–519. Springer (1985). doi: [10.1007/BFb0015776](https://doi.org/10.1007/BFb0015776)

A Details of the Semantics

We give a formal semantics for terms in Def. 9, prove that the semantics of CHPs is prefix-closed and total in Proposition 2, and prove that $d\mathcal{L}_{\text{CHP}}$ is a conservative extension of $d\mathcal{L}$ (see Proposition 1). Moreover, we prove that the operator $\hat{\circ}$ from Section 2.2 is associative.

We adapt the semantics of terms from Zwiers [42, p. 113] to our setup with real arithmetic as base terms. In order to define the semantics of terms, we use the operators on traces of the following definition:

Definition 8 (Trace operators). Let $\tau = (\tau_1, \dots, \tau_n) \in \mathcal{T}$ be a trace with communication events τ_i . Then we define its length $|\tau|$ to be n . Additionally, $|\epsilon| = 0$. For $\tau \in \mathcal{T}$ and $k \in \mathbb{N}$, we define $at(\tau, k)$ to be τ_{k+1} for $0 \leq k < |\tau|$. Otherwise, we define $at(\tau, k) = \epsilon$.

For a communication event $\rho = \langle ch, a, s \rangle$ or $\rho = \langle h, ch, a, s \rangle$, we define channel access $chan(\rho) = ch$, value access $val(\rho) = a$, and access to the timestamp $time(\rho) = s$. Moreover, we define $chan(\epsilon) = ch$ for some $ch \in \Omega$, and $val(\epsilon) = 0$, and $time(\epsilon) = 0$.

Definition 9 (Value of a term). The value $\llbracket e \rrbracket v \subseteq \mathbb{R} \cup \mathbb{N} \cup \Omega \cup \mathcal{T}$ of a term $e \in \text{Trm}(V, \Omega)$ over the state v is inductively defined in Fig. 9.

$\llbracket x \rrbracket v = v(x) \quad (x \in V_{\mathbb{R}} \cup \{\mu\})$	$\llbracket n \rrbracket v = v(n)$
$\llbracket c \rrbracket v = c$	$\llbracket \kappa \rrbracket v = \kappa \quad (\kappa \in \{0, 1\})$
$\llbracket \eta_1 + \eta_2 \rrbracket v = \llbracket \eta_1 \rrbracket v + \llbracket \eta_2 \rrbracket v$	$\llbracket ie_1 + ie_2 \rrbracket v = \llbracket ie_1 \rrbracket v + \llbracket ie_2 \rrbracket v$
$\llbracket \eta_1 \cdot \eta_2 \rrbracket v = \llbracket \eta_1 \rrbracket v \cdot \llbracket \eta_2 \rrbracket v$	$\llbracket te \rrbracket v = \llbracket te \rrbracket v $
$\llbracket \text{val}(te[ie]) \rrbracket v = val(at(\llbracket te \rrbracket v, \llbracket ie \rrbracket v))$	<p>(c) Integer terms</p>
$\llbracket \text{time}(te[ie]) \rrbracket v = time(at(\llbracket te \rrbracket v, \llbracket ie \rrbracket v))$	$\llbracket h \rrbracket v = v(h)$
<p>(a) Real terms</p>	$\llbracket \epsilon \rrbracket v = \epsilon$
$\llbracket ch \rrbracket v = ch$	$\llbracket \langle ch, \theta_1, \theta_2 \rangle \rrbracket v = \langle ch, \llbracket \theta_1 \rrbracket v, \llbracket \theta_2 \rrbracket v \rangle$
$\llbracket \text{chan}(te[ie]) \rrbracket v = chan(at(\llbracket te \rrbracket v, \llbracket ie \rrbracket v))$	$\llbracket te_1 \cdot te_2 \rrbracket v = \llbracket te_1 \rrbracket v \cdot \llbracket te_2 \rrbracket v$
<p>(b) Channel terms</p>	$\llbracket te \downarrow C \rrbracket v = (\llbracket te \rrbracket v) \downarrow C$
	<p>(d) Trace terms</p>

Fig. 9: Inductive definition of the valuation $\llbracket e \rrbracket v \subseteq \mathbb{R} \cup \mathbb{N} \cup \Omega \cup \mathcal{T}$ of a term $e \in \text{Trm}(V, \Omega)$ over the state $v \in \mathcal{S}$ [42, p. 113].

Proposition 2 (Prefix-closed and total). Let $\gamma \in \text{CHP}(V, \Omega)$ be a program. Then its semantics $\llbracket \gamma \rrbracket \subseteq \mathcal{D}$ is prefix-closed and total, i.e., if $(v, \tau, w) \in \llbracket \gamma \rrbracket$ and $(\tau', w') \preceq (\tau, w)$, then $(v, \tau', w') \in \llbracket \gamma \rrbracket$, and $\perp_{\mathcal{D}} \subseteq \llbracket \gamma \rrbracket$.

Proof. The proof is by induction on the structure of program γ . We consider α^n to be structurally smaller than α^* for all programs α and all $n \in \mathbb{N}$. W.l.o.g.

we assume $(\tau', w') \prec (\tau, w)$ in proving prefix-closedness because $(v, \tau', w') \in \llbracket \gamma \rrbracket$ trivially holds if $(v, \tau, w) \in \llbracket \gamma \rrbracket$ and $(\tau', w') = (\tau, w)$.

1. $\gamma \in \{x := \theta, x := *, ?\chi, \{x' = \theta \& \chi\}\}$, then $\llbracket \gamma \rrbracket = \perp_{\mathcal{D}} \cup U$ with $U \subseteq \mathcal{D}$. Now, let $(v, \tau, w) \in \llbracket \gamma \rrbracket$ and $(\tau', w') \prec (\tau, w)$. Then $\tau' = \epsilon$ since $\tau = \epsilon$, and $w' = \perp$. Hence, $(v, \tau', w') \in \perp_{\mathcal{D}} \subseteq \llbracket \gamma \rrbracket$. Finally, $\llbracket \gamma \rrbracket$ is total because $\perp_{\mathcal{D}} \subseteq \llbracket \gamma \rrbracket$.
2. $\gamma \in \{\text{ch}(h)! \theta, \text{ch}(h)?x\}$, then let $(v, \tau, w) \in \llbracket \gamma \rrbracket$ and $(\tau', w') \prec (\tau, w)$. Since $(v, \tau, w) \in \llbracket \gamma \rrbracket$ iff $(\tau, w) \preceq (\tilde{\tau}, \tilde{w})$ for some $(\tilde{\tau}, \tilde{w})$, we obtain $(v, \tau', w') \in \llbracket \gamma \rrbracket$ because $(\tau', w') \prec (\tau, w) \preceq (\tilde{\tau}, \tilde{w})$. Since $(\epsilon, \perp) \preceq (\tilde{\tau}, \tilde{w})$ for any $(\tilde{\tau}, \tilde{w})$, we have $(v, \epsilon, \perp) \in \llbracket \gamma \rrbracket$ for each $v \in \mathcal{S}$ such that $\llbracket \gamma \rrbracket$ is total.
3. $\gamma \equiv \alpha \cup \beta$, then $\llbracket \alpha \rrbracket$ and $\llbracket \beta \rrbracket$ are prefix-closed and total by IH. Then prefix-closedness and totality of $\llbracket \gamma \rrbracket$ follows easily from $\llbracket \gamma \rrbracket = \llbracket \alpha \rrbracket \cup \llbracket \beta \rrbracket$.
4. $\gamma \equiv \alpha; \beta$, then let $(v, \tau, w) \in \llbracket \gamma \rrbracket = \llbracket \alpha \rrbracket_{\perp} \cup \llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket$ and $(\tau', w') \prec (\tau, w)$. Thus, $w' = \perp$ and $\tau' \preceq \tau$. Observe that $\llbracket \alpha \rrbracket_{\perp}$ is prefix-closed because $\llbracket \alpha \rrbracket$ is prefix-closed by IH. So if $(v, \tau, w) \in \llbracket \alpha \rrbracket_{\perp}$, then $(v, \tau', w') \in \llbracket \alpha \rrbracket_{\perp} \subseteq \llbracket \gamma \rrbracket$. If $(v, \tau, w) \in \llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket$, then $u \neq \perp$ exists such that $(v, \tau_1, u) \in \llbracket \alpha \rrbracket$, and $(u, \tau_2, w) \in \llbracket \beta \rrbracket$, and $\tau = \tau_1 \cdot \tau_2$. If τ_1 is fully contained in τ' , i.e., $\tau' = \tau_1 \cdot \tau'_2$ for some τ'_2 , then $\tau'_2 \preceq \tau_2$ and $(\tau'_2, w') \preceq (\tau_2, w)$. Since $\llbracket \beta \rrbracket$ is prefix-closed by IH, we obtain $(u, \tau'_2, w') \in \llbracket \beta \rrbracket$, which implies $(v, \tau', w') \in \llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket \subseteq \llbracket \gamma \rrbracket$. If otherwise $\tau' \prec \tau_1 \preceq \tau$, where τ_1 is not fully contained in τ' , we have $(\tau', w') \preceq (\tau_1, u)$ since $w' = \perp$. Thus, $(v, \tau', w') \in \llbracket \alpha \rrbracket$ because $\llbracket \alpha \rrbracket$ is prefix-closed by IH. Finally, $(v, \tau', w') \in \llbracket \alpha \rrbracket_{\perp} \subseteq \llbracket \gamma \rrbracket$.
Since $\llbracket \alpha \rrbracket$ is total by IH, $\perp_{\mathcal{D}} \subseteq \llbracket \alpha \rrbracket$, which implies $\perp_{\mathcal{D}} \llbracket \alpha \rrbracket_{\perp} \subseteq \llbracket \alpha; \beta \rrbracket$.
5. $\gamma \equiv \alpha^*$, then $\llbracket \alpha^n \rrbracket$ is prefix-closed for all $n \in \mathbb{N}$ by IH as we considered α^n to be structurally smaller than α^* . This easily implies that $\llbracket \alpha^* \rrbracket = \bigcup_{n \in \mathbb{N}} \llbracket \alpha^n \rrbracket$ is prefix-closed. Finally, $\llbracket \gamma \rrbracket$ is total because $\llbracket \alpha \rrbracket \subseteq \llbracket \gamma \rrbracket$ and $\llbracket \alpha \rrbracket$ is total by IH.
6. $\gamma \equiv \alpha_1 \parallel \alpha_2$, then let $(v, \tau, w) \in \llbracket \gamma \rrbracket$ and $(\tau', w') \prec (\tau, w)$. Thus, $w' = \perp$ and $\tau' \preceq \tau$. Moreover, $(v, \tau \downarrow \alpha_j, w_{\alpha_j}) \in \llbracket \alpha_j \rrbracket$ for $j = 1, 2$, and $w_{\alpha_1}(\mu) = w_{\alpha_2}(\mu)$, and $\tau = \tau \downarrow \gamma$, and $w = w_{\alpha_1} \oplus w_{\alpha_2}$. Now, observe that $\tau' \downarrow \alpha_j \preceq \tau \downarrow \alpha_j$, which implies $(\tau' \downarrow \alpha_j, w'_{\alpha_j}) \preceq (\tau \downarrow \alpha_j, w_{\alpha_j})$ for $w'_{\alpha_j} = \perp$. Thus, $(v, \tau' \downarrow \alpha_j, w'_{\alpha_j}) \in \llbracket \alpha_j \rrbracket$ because $\llbracket \alpha_j \rrbracket$ is prefix-closed by IH. Since $\perp = \perp \oplus \perp$, we have $w' = w'_{\alpha_1} \oplus w'_{\alpha_2}$. Moreover, $\tau' = \tau' \downarrow \gamma$ and τ' is chronological as prefix $\tau' \preceq \tau$ of the chronological trace τ . Finally, $(v, \tau', w') \in \llbracket \gamma \rrbracket$.
For each $v \in \mathcal{S}$ and $j = 1, 2$, some $(v, \epsilon, \perp) \in \llbracket \alpha_j \rrbracket$ since $\llbracket \alpha_j \rrbracket$ is total by IH. Then $\llbracket \gamma \rrbracket$ is total because $(v, \epsilon, \perp) \in \llbracket \gamma \rrbracket$ for each $v \in \mathcal{S}$ since $\perp(\mu) = \perp(\mu)$, and $\epsilon = \epsilon \downarrow \gamma$, and $\perp = \perp \oplus \perp$. \square

In preparation for the proof of conservative extension (Proposition 1), we roughly recap $d\mathcal{L}$ in the following [30]:

Remark 2 ($d\mathcal{L}$). Terms $\text{Trm}_{d\mathcal{L}} = \mathbb{Q}[V_{\mathbb{R}}] \subset \text{Trm}(V, \Omega)$ in $d\mathcal{L}$ are the polynomials in $V_{\mathbb{R}}$ over \mathbb{Q} . Moreover, hybrid programs $\text{HP}_{d\mathcal{L}} \subset \text{CHP}(V, \Omega)$ in $d\mathcal{L}$ are generated by the following grammar: $\alpha, \beta ::= x := \theta \mid x := * \mid ?\chi \mid \{x' = \theta \& \chi\} \mid \alpha; \beta \mid \alpha \cup \beta \mid \alpha^*$, where $\theta \in \text{Trm}_{d\mathcal{L}}$ and $\chi \in \text{FOL}_{\mathbb{R}}(V_{\mathbb{R}})$ is a formula of first-order real-arithmetic. Finally, $\text{Fml}_{d\mathcal{L}}$ denotes the set of all formulas in $d\mathcal{L}$ that is generated by the grammar $\varphi, \psi ::= \theta_1 \sim \theta_2 \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi \mid \forall x \varphi \mid \exists x \varphi \mid [\alpha]\psi \mid \langle \alpha \rangle \psi$, where $\sim \in \{=, \geq\}$. Note that $\text{FOL}_{\mathbb{R}}(V_{\mathbb{R}}) \subset \text{Fml}_{d\mathcal{L}}$.

A real state is a map from $V_{\mathbb{R}}$ to \mathbb{R} and \mathcal{R} denotes the set of all real states. With $\llbracket \cdot \rrbracket_{d\mathcal{L}} \subseteq \mathcal{R}$ we denote semantics of terms and with $\models_{d\mathcal{L}} \subseteq \mathcal{R} \times \text{Fml}_{d\mathcal{L}}$ the satisfaction relation in $d\mathcal{L}$, respectively. The semantics $\llbracket \alpha \rrbracket_{d\mathcal{L}} \subseteq \mathcal{R} \times \mathcal{R}$ of a hybrid program $\alpha \in \text{HP}_{d\mathcal{L}}$ is inductively defined as follows:

$$\begin{aligned} \llbracket x := \theta \rrbracket_{d\mathcal{L}} &= \{(v, w) \in \mathcal{R} \times \mathcal{R} \mid w = v_x^{\llbracket \theta \rrbracket_{d\mathcal{L}} v}\} \\ \llbracket x := * \rrbracket_{d\mathcal{L}} &= \{(v, w) \in \mathcal{R} \times \mathcal{R} \mid w(y) = v(y) \text{ for all } y \neq x\} \\ \llbracket \{x' = \theta \ \& \ \chi\} \rrbracket_{d\mathcal{L}} &= \{(\varphi(0), \varphi(r)) \in \mathcal{R} \times \mathcal{R} \mid \\ &\quad \varphi(\zeta) \models_{d\mathcal{L}} x' = \theta \wedge \chi \text{ and } \varphi(\zeta) = \varphi(0) \text{ on } \{x\}^{\mathbb{G}} \text{ for all } \zeta \in [0, r] \\ &\quad \text{for a solution } \varphi : [0, r] \rightarrow \mathcal{R} \text{ with } \varphi(\zeta)(x') = \frac{d\varphi(t)(x)}{dt}(\zeta)\} \\ \llbracket ?\chi \rrbracket_{d\mathcal{L}} &= \{(v, v) \in \mathcal{R} \times \mathcal{R} \mid v \models_{d\mathcal{L}} \chi\} \\ \llbracket \alpha \cup \beta \rrbracket_{d\mathcal{L}} &= \llbracket \alpha \rrbracket_{d\mathcal{L}} \cup \llbracket \beta \rrbracket_{d\mathcal{L}} \\ \llbracket \alpha ; \beta \rrbracket_{d\mathcal{L}} &= \llbracket \alpha \rrbracket_{d\mathcal{L}} \circ \llbracket \beta \rrbracket_{d\mathcal{L}} \quad (\circ \text{ is composition of relations}) \\ \llbracket \alpha^* \rrbracket_{d\mathcal{L}} &= \bigcup_{n \in \mathbb{N}} \llbracket \alpha^n \rrbracket_{d\mathcal{L}} \end{aligned}$$

For a state $v \in \mathcal{S}$, the real state $v|_{V_{\mathbb{R}}} \in \mathcal{R}$ is the restriction of v to $V_{\mathbb{R}}$. The restriction $U|_{\mathcal{R} \times \mathcal{R}}$ of a set $U \subseteq \mathcal{S} \times \mathcal{T}_{\text{rec}} \times \mathcal{S}_{\perp}$ to $\mathcal{R} \times \mathcal{R}$ is defined by

$$U|_{\mathcal{R} \times \mathcal{R}} = \{(v|_{V_{\mathbb{R}}}, w|_{V_{\mathbb{R}}}) \in \mathcal{R} \times \mathcal{R} \mid (v, \tau, w) \in U \text{ with } w \neq \perp\}.$$

In preparation for the proof of Proposition 1, Lemma 1 states that $d\mathcal{L}_{\text{CHP}}$ is conservative extension of $d\mathcal{L}$ w.r.t. terms and formulas in hybrid programs $\text{HP}_{d\mathcal{L}}$. Building on this Lemma 2 shows that the semantics of CHPs is a conservative extension of the program semantics in $d\mathcal{L}$. We continue to denote the semantics of terms and programs in $d\mathcal{L}_{\text{CHP}}$ by $\llbracket \cdot \rrbracket$ and the satisfaction relation by \models .

Lemma 1 (Conservative extension of program parts). *Let $\theta \in \text{Trm}_{d\mathcal{L}}$ be a $d\mathcal{L}$ -term and $\chi \in \text{FOL}_{\mathbb{R}}(V_{\mathbb{R}})$ a formula of first-order real arithmetic. Then for all states $v \in \mathcal{S}$ the following holds:*

1. $\llbracket \theta \rrbracket_{d\mathcal{L}} v|_{V_{\mathbb{R}}} = \llbracket \theta \rrbracket v$
2. $v|_{V_{\mathbb{R}}} \models_{d\mathcal{L}} \chi$ iff $v \models \chi$

Proof. The proof is by induction on the structure of θ and χ , respectively. \square

Lemma 2 (Conservative program semantics). *The program semantics of $d\mathcal{L}_{\text{CHP}}$ is a conservative extension of the program semantics in $d\mathcal{L}$. That is,*

$$\llbracket \gamma \rrbracket_{d\mathcal{L}} = \llbracket \gamma \rrbracket|_{\mathcal{R} \times \mathcal{R}} = \{(v|_{V_{\mathbb{R}}}, w|_{V_{\mathbb{R}}}) \mid (v, \tau, w) \in \llbracket \gamma \rrbracket \text{ with } w \neq \perp\}$$

for a hybrid program $\gamma \in \text{HP}_{d\mathcal{L}} \subset \text{CHP}(V, \Omega)$.

Proof. The proof is by induction on the structure of program γ . We consider α^n to be structurally smaller than α^* for all programs α and all $n \in \mathbb{N}$.

1. For $\gamma \in \{x := \theta, x := *, ?\chi, \{x' = \theta \ \& \ \chi\}\}$, we have $\llbracket \gamma \rrbracket = \perp_{\mathcal{D}} \cup M_\gamma$ for some $M_\gamma \subseteq \mathcal{S} \times \{\epsilon\} \times \mathcal{S}$. Observe that $\llbracket \gamma \rrbracket|_{\mathcal{R} \times \mathcal{R}} = M_\gamma|_{\mathcal{R} \times \mathcal{R}}$ because $\perp_{\mathcal{D}} = \mathcal{S} \times \{\epsilon\} \times \{\perp\}$. Finally, $\llbracket \gamma \rrbracket_{d\mathcal{L}} = M_\gamma|_{\mathcal{R} \times \mathcal{R}}$ by definition of $\llbracket \gamma \rrbracket_{d\mathcal{L}}$ and M_γ . For instance, in case $\gamma \equiv x := \theta$, we conclude with $M_\gamma|_{\mathcal{R} \times \mathcal{R}} = \{(v, \epsilon, w) \mid w = v_x^{\llbracket \theta \rrbracket v}\}|_{\mathcal{R} \times \mathcal{R}} = \{(v|_{V_{\mathbb{R}}}, w|_{V_{\mathbb{R}}}) \mid w = v_x^{\llbracket \theta \rrbracket v}\} = \{(v|_{V_{\mathbb{R}}}, w|_{V_{\mathbb{R}}}) \mid w|_{V_{\mathbb{R}}} = (v|_{V_{\mathbb{R}}})_x^{\llbracket \theta \rrbracket_{d\mathcal{L}} v|_{V_{\mathbb{R}}}}\} = \llbracket x := \theta \rrbracket_{d\mathcal{L}}$ because $\llbracket \theta \rrbracket v = \llbracket \theta \rrbracket_{d\mathcal{L}} v|_{V_{\mathbb{R}}}$ by Lemma 1.
2. For $\gamma \equiv \alpha \cup \beta$, observe that $\llbracket \alpha \rrbracket|_{\mathcal{R} \times \mathcal{R}} \cup \llbracket \beta \rrbracket|_{\mathcal{R} \times \mathcal{R}} = (\llbracket \alpha \rrbracket \cup \llbracket \beta \rrbracket)|_{\mathcal{R} \times \mathcal{R}}$. Then by IH, $\llbracket \gamma \rrbracket_{d\mathcal{L}} = \llbracket \alpha \rrbracket_{d\mathcal{L}} \cup \llbracket \beta \rrbracket_{d\mathcal{L}} \stackrel{\text{IH}}{=} \llbracket \alpha \rrbracket|_{\mathcal{R} \times \mathcal{R}} \cup \llbracket \beta \rrbracket|_{\mathcal{R} \times \mathcal{R}} = \llbracket \gamma \rrbracket|_{\mathcal{R} \times \mathcal{R}}$.
3. For $\gamma \equiv \alpha; \beta$, we have $\llbracket \gamma \rrbracket|_{\mathcal{R} \times \mathcal{R}} = \llbracket \alpha \rrbracket_{\perp}|_{\mathcal{R} \times \mathcal{R}} \cup (\llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket)|_{\mathcal{R} \times \mathcal{R}}$ since restriction $\cdot|_{\mathcal{R} \times \mathcal{R}}$ distributes over union \cup . Observe that $\llbracket \alpha \rrbracket_{\perp}|_{\mathcal{R} \times \mathcal{R}} = \emptyset$ and $(\llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket)|_{\mathcal{R} \times \mathcal{R}} = \llbracket \alpha \rrbracket|_{\mathcal{R} \times \mathcal{R}} \circ \llbracket \beta \rrbracket|_{\mathcal{R} \times \mathcal{R}}$, where \circ is composition of relations. Thus, $\llbracket \gamma \rrbracket_{d\mathcal{L}} = \llbracket \alpha \rrbracket_{d\mathcal{L}} \circ \llbracket \beta \rrbracket_{d\mathcal{L}} \stackrel{\text{IH}}{=} \llbracket \alpha \rrbracket|_{\mathcal{R} \times \mathcal{R}} \circ \llbracket \beta \rrbracket|_{\mathcal{R} \times \mathcal{R}} = \llbracket \gamma \rrbracket|_{\mathcal{R} \times \mathcal{R}}$ by IH.
4. $\gamma \equiv \alpha^*$, then $(v, w) \in \llbracket \alpha^* \rrbracket_{d\mathcal{L}}$ iff $(v, w) \in \llbracket \alpha^n \rrbracket_{d\mathcal{L}}$ for some $n \in \mathbb{N}$ iff, by IH, $(v, w) \in \llbracket \alpha^n \rrbracket|_{\mathcal{R} \times \mathcal{R}}$ for some $n \in \mathbb{N}$ iff $(v, w) \in \llbracket \alpha^* \rrbracket|_{\mathcal{R} \times \mathcal{R}}$. \square

Proof (Proposition 1). We have to prove that a formula $\varphi \in \text{Fml}(V, \Omega) \cap \text{Fml}_{d\mathcal{L}}$ is valid in $d\mathcal{L}_{\text{CHP}}$ iff it is valid in $d\mathcal{L}$, where φ is valid in $d\mathcal{L}$ if $v|_{V_{\mathbb{R}}} \models_{d\mathcal{L}} \varphi$ for all $v \in \mathcal{S}$, and φ is valid in $d\mathcal{L}_{\text{CHP}}$ if $v \models \varphi$ for all $v \in \mathcal{S}$. The proof is by induction on the structure of φ :

1. $\varphi \equiv \theta_1 \sim \theta_2$, where $\sim \in \{=, \geq\}$, then $v \models \theta_1 \sim \theta_2$ iff $\llbracket \theta_1 \rrbracket v \sim \llbracket \theta_2 \rrbracket v$ iff, by Lemma 1, $\llbracket \theta_1 \rrbracket_{d\mathcal{L}} v|_{V_{\mathbb{R}}} \sim \llbracket \theta_2 \rrbracket_{d\mathcal{L}} v|_{V_{\mathbb{R}}}$ iff $v|_{V_{\mathbb{R}}} \models_{d\mathcal{L}} e_1 \sim e_2$.
2. For the propositional connectives $\neg, \wedge, \vee, \rightarrow$ and quantifiers \forall, \exists the proof is straightforward by IH.
3. For $\varphi \equiv [\alpha]\psi$, observe $(v|_{V_{\mathbb{R}}}, w|_{V_{\mathbb{R}}}) \in \llbracket \alpha \rrbracket_{d\mathcal{L}}$ iff $(v, \tau, w) \in \llbracket \alpha \rrbracket$ with $w \neq \perp$ by Lemma 2 and $\tau = \epsilon$ since $\alpha \in \text{CHP}(V, \Omega)$. Thus, $v \models [\alpha]\psi$ iff $w \cdot \tau \models \psi$ for all $(v, \tau, w) \in \llbracket \alpha \rrbracket$ with $w \cdot \tau = w \neq \perp$ iff, by IH, $w|_{V_{\mathbb{R}}} \models_{d\mathcal{L}} \psi$ for all $(v, \tau, w) \in \llbracket \alpha \rrbracket$ with $w \neq \perp$ iff $w|_{V_{\mathbb{R}}} \models_{d\mathcal{L}} \psi$ for all $(v|_{V_{\mathbb{R}}}, w|_{V_{\mathbb{R}}}) \in \llbracket \alpha \rrbracket_{d\mathcal{L}}$ iff $v|_{V_{\mathbb{R}}} \models_{d\mathcal{L}} [\alpha]\psi$. \square

Lemma 3 shows that the composition operator $\hat{\circ}$ is associative. See Section 2.2 for the definition of the operators involved.

Lemma 3 (Composition is associative). *Let $U, M, L \subseteq \mathcal{S} \times \mathcal{T}_{\text{rec}} \times \mathcal{S}_{\perp}$. Then $U \hat{\circ} (M \hat{\circ} L) = (U \hat{\circ} M) \hat{\circ} L$.*

Proof. Let $(v, \tau, w) \in U \hat{\circ} (M \hat{\circ} L)$. If $(v, \tau, w) \in U_{\perp}$, then $(v, \tau, w) \in U_{\perp} \cup (U \triangleright M)_{\perp} \subseteq (U \hat{\circ} M)_{\perp} \subseteq (U \hat{\circ} M) \hat{\circ} L$. If $(v, \tau, w) \in U \triangleright (M \hat{\circ} L)$, computations $(v, \tau_1, u) \in U$ and $(u, \tau_2, w) \in (M \hat{\circ} L)$ with $\tau = \tau_1 \cdot \tau_2$ exist. Now, if $(u, \tau_2, w) \in M_{\perp}$, we have $(v, \tau, w) \in U \triangleright M_{\perp} = (U \triangleright M)_{\perp} \subseteq (U \hat{\circ} M)_{\perp}$. If $(u, \tau_2, w) \in M \triangleright L$, then $(v, \tau, w) \in U \triangleright (M \triangleright L)$. Since \triangleright is associative, $(v, \tau, w) \in (U \triangleright M) \triangleright L \subseteq (U \hat{\circ} M) \triangleright L \subseteq (U \hat{\circ} M) \hat{\circ} L$.

Conversely, let $(v, \tau, w) \in (U \hat{\circ} M) \hat{\circ} L$. If $(v, \tau, w) \in (U \hat{\circ} M)_{\perp}$, then $(v, \tau, w) \in U_{\perp}$ or $(v, \tau, w) \in (U \triangleright M)_{\perp}$. If $(v, \tau, w) \in U_{\perp}$, we conclude $U_{\perp} \subseteq U \hat{\circ} (M \hat{\circ} L)$. If $(v, \tau, w) \in (U \triangleright M)_{\perp}$, computations $(v, \tau_1, u) \in U$ and $(u, \tau_2, w) \in M_{\perp} \subseteq M \hat{\circ} L$ with $\tau = \tau_1 \cdot \tau_2$ exist. Thus, $(v, \tau, w) \in U \triangleright (M \hat{\circ} L) \subseteq U \hat{\circ} (M \hat{\circ} L)$. Finally, if $(v, \tau, w) \in (U \hat{\circ} M) \triangleright L$, computations $(v, \tau_1, u) \in U \hat{\circ} M$ and $(u, \tau_2, w) \in L$ with

$\tau = \tau_1 \cdot \tau_2$ exist. Since $u \neq \perp$, $(v, \tau_1, u) \in U \triangleright M$ such that $(v, \tau, w) \in (U \triangleright M) \triangleright L$. Thus, $(v, \tau, w) \in U \triangleright (M \triangleright L) \subseteq U \triangleright (M \hat{\circ} L) \subseteq U \hat{\circ} (M \hat{\circ} L)$ because \triangleright is associative. \square

B Static Semantics

The static semantics captures with free variables $FV(\cdot)$, which variables potentially influence terms, programs, or formulas, and with bound variables $BV(\cdot)$, which variables are written by a program. Following the static semantics of $d\mathcal{L}$ [33] in general, we give more precise coincidence properties, which take the communication history into account. Therefore, we adapt the notions of accessed channels and communication-aware coincidence for terms from Zwiers [42] and lift them to CHPs and formulas of dynamic logic.

Remark 3 (Communication-awareness). Communication-aware coincidence reflects that the portion of a communication trace $v(h)$ a formula over trace variable h depends on shrinks with projections of h . For example, $|h| > 0$ depends on the full trace $v(h)$ but $|h \downarrow \text{ch}| > 0$ only on $v(h) \downarrow \text{ch}$. The sets of accessed channels $CN(e)$, and $CN(\varphi)$ in terms e and formulas φ collect the relevant channels of all communication traces $v(h)$ with $h \in V_{\mathcal{T}}$.

Definition 10 (Bound variables). *The set of bound variables $BV(\gamma)$ of a program $\gamma \in \text{CHP}(V, \Omega)$ is inductively defined in Fig. 10.*

$$\begin{aligned}
 BV(x := e) &= BV(x := *) = \{x\} \\
 BV(\{x' = \theta \ \& \ \chi\}) &= \{x, \mu\} \\
 BV(?\chi) &= \emptyset \\
 BV(\text{ch}(h)! \theta) &= \{h\} \\
 BV(\text{ch}(h)?x) &= \{h, x\} \\
 BV(\alpha \cup \beta) &= BV(\alpha; \beta) = BV(\alpha \parallel \beta) = BV(\alpha) \cup BV(\beta) \\
 BV(\alpha^*) &= BV(\alpha)
 \end{aligned}$$

Fig. 10: Inductive definition of the set of bound variables $BV(\gamma)$ of a program $\gamma \in \text{CHP}(V, \Omega)$.

Def. 10 is as usual except that continuous evolution silently (no syntactic occurrence) binds the global time μ since it evolves with every ODE. The bound effect property in Lemma 4 takes into account that the worlds reachable by a program consist of the final state and the communication as opposed to only considering the final state.

Lemma 4 (Bound effect property). *The set of bound variables $BV(\gamma)$ has the bound effect property for a program $\gamma \in \text{CHP}(V, \Omega)$. That is, $v = w$ on $BV(\gamma)^{\mathbb{G}} \cup V_{\mathcal{T}}$ and $v = w \cdot \tau$ on $BV(\gamma)^{\mathbb{G}}$ for all $(v, \tau, w) \in \llbracket \gamma \rrbracket$ with $w \neq \perp$.*

Proof. Let $REC(\gamma) \subseteq V_{\mathcal{T}}$ be the set of recorder variables in γ . Then $\tau(h) = \epsilon$ for all $h \notin REC(\gamma)$ by an induction on the structure of γ , where $\tau(h)$ is the subtrace of τ of communication recorded by h . Hence, $w = w \cdot \tau$ on $REC(\gamma)^{\mathbb{G}}$. Since $REC(\gamma) \subseteq BV(\gamma)$, we obtain $v = w = w \cdot \tau$ on $BV(\gamma)^{\mathbb{G}} \cap REC(\gamma)^{\mathbb{G}} = (BV(\gamma) \cup REC(\gamma))^{\mathbb{G}} = BV(\gamma)^{\mathbb{G}}$ if only $v = w$ on $BV(\gamma)^{\mathbb{G}}$. The latter also shows $v = w$ on $BV(\alpha)^{\mathbb{G}} \cup V_{\mathcal{T}}$ because $v = w$ on $V_{\mathcal{T}}$ by another induction on γ . Now, we prove $v = w$ on $BV(\gamma)^{\mathbb{G}}$ by induction on the structure of γ . We consider α^n to be structurally smaller than α^* for all programs α and all $n \in \mathbb{N}$.

1. $\gamma \in \{x := \theta, x := *, \text{ch}(h)?x\}$ and $(v, \tau, w) \in \llbracket \gamma \rrbracket$, then $w = v_x^a$ for some $a \in \mathbb{R}$. Thus, $v = w$ on $\{x\}^{\mathbb{G}} = BV(\gamma)^{\mathbb{G}}$.
2. $\gamma \equiv \{x' = \theta \ \& \ \chi\}$ and $(v, \tau, w) \in \llbracket \gamma \rrbracket$, then a solution $\varphi : [0, s] \rightarrow \mathcal{S}$ exists with $\varphi(\zeta) = v$ on $\{x, \mu\}^{\mathbb{G}}$ for all $\zeta \in [0, s]$ and $w = \varphi(s)$. Thus, $v = w$ on $\{x, \mu\}^{\mathbb{G}} = BV(\gamma)^{\mathbb{G}}$.
3. $\gamma \in \{?\chi, \text{ch}(h)!\theta\}$ and $(v, \tau, w) \in \llbracket \gamma \rrbracket$, then $v = w$. Thus, $v = w$ on $BV(\gamma)^{\mathbb{G}}$.
4. $\gamma \equiv \alpha; \beta$ and $(v, \tau, w) \in \llbracket \gamma \rrbracket$, then $(v, \tau, w) \in \llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket \subseteq \llbracket \gamma \rrbracket$. Thus, $u \neq \perp$ exists such that $(v, \tau_1, u) \in \llbracket \alpha \rrbracket$, $(u, \tau_2, w) \in \llbracket \beta \rrbracket$, and $\tau = \tau_1 \cdot \tau_2$. By IH, $v = u$ on $BV(\alpha)^{\mathbb{G}}$ and $u = w$ on $BV(\beta)^{\mathbb{G}}$. Thus, $v = w$ on $BV(\alpha)^{\mathbb{G}} \cap BV(\beta)^{\mathbb{G}} = (BV(\alpha) \cup BV(\beta))^{\mathbb{G}} = BV(\alpha; \beta)^{\mathbb{G}}$.
5. $\gamma \equiv \alpha \cup \beta$ and $(v, \tau, w) \in \llbracket \gamma \rrbracket$, then $(v, \tau, w) \in \llbracket \alpha \rrbracket$ or $(v, \tau, w) \in \llbracket \beta \rrbracket$. By IH, $v = w$ on $BV(\alpha)^{\mathbb{G}}$ or on $BV(\beta)^{\mathbb{G}}$, respectively. Always, $v = w$ on $BV(\alpha)^{\mathbb{G}} \cap BV(\beta)^{\mathbb{G}} = (BV(\alpha) \cup BV(\beta))^{\mathbb{G}} = BV(\alpha \cup \beta)^{\mathbb{G}}$.
6. $\gamma \equiv \alpha^*$ and $(v, \tau, w) \in \llbracket \gamma \rrbracket$, then $(v, \tau, w) \in \llbracket \alpha^n \rrbracket$ for some $n \in \mathbb{N}$. Since α^n is structurally smaller than α^* , we obtain $v = w$ on $BV(\alpha^n)^{\mathbb{G}}$ by IH. Finally, $BV(\alpha^n)^{\mathbb{G}} \supseteq BV(\alpha)^{\mathbb{G}} = BV(\alpha^*)^{\mathbb{G}}$ (note that $\alpha^0 \equiv ?\top$).
7. $\gamma \equiv \alpha \parallel \beta$ and $(v, \tau, w) \in \llbracket \gamma \rrbracket$, then $(v, \tau \downarrow \alpha, w_\alpha) \in \llbracket \alpha \rrbracket$, $(v, \tau \downarrow \beta, w_\beta) \in \llbracket \beta \rrbracket$, and $w = w_\alpha \oplus w_\beta$. By IH, $v = w_\beta$ on $BV(\beta)^{\mathbb{G}}$. Moreover, $w_\beta = w$ on $BV(\alpha)^{\mathbb{G}}$ by definition of \oplus in Section 2.2. Thus, $v = w$ on $BV(\alpha)^{\mathbb{G}} \cap BV(\beta)^{\mathbb{G}} = (BV(\alpha) \cup BV(\beta))^{\mathbb{G}} = BV(\alpha \parallel \beta)^{\mathbb{G}}$. \square

Definition 11 (Parameters of terms). *The sets of free variables $FV(e)$ and accessed channels $CN(e)$ of a term $e \in \text{Trm}(V, \Omega)$ are inductively defined in Fig. 11.*

Unsurprisingly, a trace variable h potentially accesses all channels Ω by Def. 11, and with a projection on C the accessed channels shrink by C . The communication item $\langle \text{ch}, \theta_1, \theta_2 \rangle$ does not access any channels since $\theta_1, \theta_2 \in \text{FOL}_{\mathbb{R}}(V_{\mathbb{R}})$.

For use in the coincide properties, we lift projection for traces to states by defining that $v \downarrow C = v$ on $V_{\mathbb{R}} \cup V_{\mathbb{N}}$ and $(v \downarrow C)(h) = v(h) \downarrow C$ for all $h \in V_{\mathcal{T}}$ and $C \subseteq \Omega$. As for traces, we often write $v \downarrow e$ instead of $v \downarrow CN(e)$.

Lemma 5 (Coincidence for terms). *The pair of free variables $FV(e)$ and accessed channels $CN(e)$ has the communication-aware coincidence property for a term $e \in \text{Trm}(V, \Omega)$. That is, if $v \downarrow e = \tilde{v} \downarrow e$ on $FV(e)$, then $\llbracket e \rrbracket v = \llbracket e \rrbracket \tilde{v}$. In particular, $\llbracket e \rrbracket v = \llbracket e \rrbracket \tilde{v}$ if $v = \tilde{v}$ on $FV(e)$.*

Proof. W.l.o.g. we push down projections in e as far as possible by applying the axioms $\epsilon \downarrow$, $\cdot \downarrow$, and $\downarrow \cap$, and $\downarrow \in$, and $\downarrow \notin$ (see Fig. 15) from left to right.

$$\begin{array}{ll}
FV(z) = \{z\} \quad (z \in V) & CN(h) = \Omega \quad (h \in V_{\mathcal{T}}) \\
FV(\kappa) = \emptyset & CN(z) = CN(\kappa) = \emptyset \quad (z \notin V_{\mathcal{T}}) \\
FV(e_1 \bowtie e_2) = FV(e_1) \cup FV(e_2) & CN(e_1 \bowtie e_2) = CN(e_1) \cup CN(e_2) \\
FV(f(te, ie)) = FV(te) \cup FV(ie) & CN(f(te, ie)) = CN(te) \cup CN(ie) \\
FV(|te|) = FV(te) & CN(|te|) = CN(te) \\
FV(\langle \text{ch}, \theta_1, \theta_2 \rangle) = FV(\theta_1) \cup FV(\theta_2) & CN(\langle \text{ch}, \theta_1, \theta_2 \rangle) = \emptyset \\
FV(te \downarrow C) = FV(te) & CN(te \downarrow C) = CN(te) \cap C
\end{array}$$

Fig. 11: Inductive definition of free variables $FV(e)$ and accessed channels $CN(e)$ of a term $e \in \text{Trm}(V, \Omega)$, where $z \in V$, κ is a constant of any sort, $\bowtie \in \{+, \cdot\}$ is any operator, and $f(te, ie) \in \{\text{chan}(te[ie]), \text{val}(te[ie]), \text{time}(te[ie])\}$ is a term over te and ie .

This is justified since valuation $\llbracket e \rrbracket$, free variables $FV(e)$, and accessed channels $CN(e)$ are invariant under this rewritings. In the resulting normal form only raw trace variables occur in the scope of projections. Crucially, the communication item $\langle \text{ch}, \theta_1, \theta_2 \rangle$ does not contain further trace variables since $\theta_1, \theta_2 \in \text{FOI}_{\mathbb{R}}(V_{\mathbb{R}})$. Now, the proof is by induction on the structure of the term e :

1. $e \equiv z$ with $z \in V$, then $FV(e) = \{z\}$. If $z \in V_{\mathbb{R} \cup \mathbb{N}} \cup \{\mu\}$, then $v(z) = \tilde{v}(z)$ by premise. If $z \in V_{\mathcal{T}}$, then $v(z) = v(z) \downarrow e = \tilde{v}(z) \downarrow e = \tilde{v}(z)$ since $CN(e) = \Omega$ and $v \downarrow e = \tilde{v} \downarrow e$ on $\{z\}$ by premise. Thus, $\llbracket e \rrbracket v = v(z) = \tilde{v}(z) = \llbracket e \rrbracket \tilde{v}$.
2. $e \equiv \kappa$, where κ is a constant of any sort, then $\llbracket e \rrbracket v = \kappa = \llbracket e \rrbracket \tilde{v}$.
3. $e \equiv e_1 \bowtie e_2$ with $\bowtie \in \{+, \cdot\}$, then $v \downarrow e = \tilde{v} \downarrow e$ on $FV(e_j)$ since $FV(e_j) \subseteq FV(e)$. For $h \in V_{\mathcal{T}}$, observe that $v(h) \downarrow D = \tilde{v}(h) \downarrow D$ implies $v(h) \downarrow E = \tilde{v}(h) \downarrow E$ if $E \subseteq D \subseteq \Omega$. Hence, we obtain $v \downarrow e_j = \tilde{v} \downarrow e_j$ on $FV(e_j)$ from $CN(e_j) \subseteq CN(e)$. By IH, we conclude as follows:

$$\llbracket e \rrbracket v = \llbracket e_1 \rrbracket v \bowtie \llbracket e_2 \rrbracket v \stackrel{\text{IH}}{=} \llbracket e_1 \rrbracket \tilde{v} \bowtie \llbracket e_2 \rrbracket \tilde{v} = \llbracket e \rrbracket \tilde{v}$$

4. $e \in \{\text{chan}(te[ie]), \text{val}(te[ie]), \text{time}(te[ie]), \langle \text{ch}, \theta_1, \theta_2 \rangle, |te|\}$, then we conclude similar to the last case by IH.
5. $e \equiv te \downarrow C$, then $te \equiv h$ for some $h \in V_{\mathcal{T}}$ since e is in normal form by assumption. From $CN(e) = C$ and $FV(e) = \{h\}$, we obtain $v(h) \downarrow C = \tilde{v}(h) \downarrow C$ by premise such that $\llbracket e \rrbracket v = v(h) \downarrow C = \tilde{v}(h) \downarrow C = \llbracket e \rrbracket \tilde{v}$. \square

We adapt must-bound variables from $d\mathcal{L}$ [33] leading to a fine-grained definition of free variables in formulas and programs. Defining $FV([\alpha]\psi)$ as $FV(\alpha) \cup FV(\psi)$ would be sound w.r.t. the coincidence property for formulas but imprecise because $[x := 0]x \geq 0$ does not depend on x . However, defining $FV([\alpha]\psi)$ as $FV(\alpha) \cup (FV(\psi) \setminus BV(\alpha))$ is unsound because $[x := 0 \cup y := 0]x \geq 0$ depends on x as x is not bound on all execution paths. The must-bound variables are the variables that are bound on all execution paths of a program. Hence, they may be removed from $FV(\psi)$ soundly in $FV([\alpha]\psi) = FV(\alpha) \cup (FV(\psi) \setminus MBV(\alpha))$.

Definition 12 (Must-bound variables). *The set of must-bound variables $MBV(\gamma)$ of a program $\gamma \in \text{CHP}(V, \Omega)$ is inductively defined in Fig. 12.*

$$\begin{aligned}
 MBV(\alpha) &= BV(\alpha) \quad (\alpha \text{ is any atomic program}) \\
 MBV(\alpha \cup \beta) &= MBV(\alpha) \cap MBV(\beta) \\
 MBV(\alpha; \beta) &= MBV(\alpha \parallel \beta) = MBV(\alpha) \cup MBV(\beta) \\
 MBV(\alpha^*) &= \emptyset
 \end{aligned}$$

Fig. 12: Inductive definition of the must-bound variables $MBV(\gamma)$ of a program $\gamma \in \text{CHP}(V, \Omega)$.

Definition 13 (Free variables of programs). *The set of free variables $FV(\gamma)$ of a program $\gamma \in \text{CHP}(V, \Omega)$ is inductively defined in Fig. 13.*

$$\begin{aligned}
 FV(x := \theta) &= FV(\theta) \\
 FV(x := *) &= \emptyset \\
 FV(?\chi) &= FV(\chi) \\
 FV(\{x' = \theta \& \chi\}) &= \{x, \mu\} \cup FV(\theta) \cup FV(\chi) \\
 FV(\text{ch}(h)!\theta) &= \{h, \mu\} \cup FV(\theta) \\
 FV(\text{ch}(h)?x) &= \{h, \mu\} \\
 FV(\alpha; \beta) &= FV(\alpha) \cup (FV(\beta) \setminus MBV(\alpha)) \\
 FV(\alpha \cup \beta) &= FV(\alpha \parallel \beta) = FV(\alpha) \cup FV(\beta) \\
 FV(\alpha^*) &= FV(\alpha)
 \end{aligned}$$

Fig. 13: Inductive definition of free variables $FV(\gamma)$ of a program $\gamma \in \text{CHP}(V, \Omega)$.

Def. 13 is as usual except that a continuous evolution silently (no syntactic occurrence) depends on the global time μ as it evolves like x with the ODE. Moreover, μ is free in $\text{ch}(h)!\theta$ and $\text{ch}(h)?x$ because the current time is recorded as timestamp for each communication event.

Definition 14 (Free parameters of formulas). *The sets of free variables $FV(\phi)$ and accessed channels $CN(\phi)$ of a formula $\phi \in \text{Fml}(V, \Omega)$ is inductively defined in Fig. 14.*

Lemma 6 (Coincidence for formulas). *The pair of free variables $FV(\phi)$ and accessed channels $CN(\phi)$ has the communication-aware coincidence property for a formula $\phi \in \text{Fml}(V, \Omega)$: If $v \downarrow \phi = \tilde{v} \downarrow \phi$ on $FV(\phi)$, then: $v \in \llbracket \phi \rrbracket$ iff $\tilde{v} \in \llbracket \phi \rrbracket$.*

Lemma 7 (Coincidence for programs). *The set of free variables $FV(\gamma)$ has the communication-aware coincidence property for a program $\gamma \in \text{CHP}(V, \Omega)$: If $v \downarrow C = \tilde{v} \downarrow C$ on $X \supseteq FV(\gamma)$ with $C \subseteq \Omega$ and $(v, \tau, w) \in \llbracket \gamma \rrbracket$, then $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \gamma \rrbracket$ exists with $w \downarrow C = \tilde{w} \downarrow C$ on $X \cup MBV(\gamma)$, and $\tau = \tilde{\tau}$, and $(w = \perp$ iff $\tilde{w} = \perp)$.*

Proof (Lemma 6 and Lemma 7). We generalize the proof of the coincidence properties for $d\mathcal{L}$ [33, Lemma 17] to CHPs and formulas featuring ac-modalities. The proof is by simultaneous induction on the structure of formulas and programs.

$$\begin{array}{ll}
FV(e_1 \sim e_2) = FV(e_1) \cup FV(e_2) & CN(e_1 \sim e_2) = CN(e_1) \cup CN(e_2) \\
FV(\neg\varphi) = FV(\varphi) & CN(\neg\varphi) = CN(\varphi) \\
FV(\varphi \Rightarrow \psi) = FV(\varphi) \cup FV(\psi) & CN(\varphi \Rightarrow \psi) = CN(\varphi) \cup CN(\psi) \\
FV(\mathcal{Q}z. \varphi) = FV(\varphi) \setminus \{z\} & CN(\mathcal{Q}z. \varphi) = CN(\varphi) \\
FV([\alpha]\psi) = FV(\alpha) \cup (FV(\psi) \setminus MBV(\alpha)) & CN([\alpha]\psi) = CN(\psi) \\
FV([\alpha]_{\{A,C\}}\psi) = FV([\alpha]\psi) \cup FV(A) \cup FV(C) & CN([\alpha]_{\{A,C\}}\psi) = CN([\alpha]\psi) \cup CN(A) \cup CN(C)
\end{array}$$

Fig. 14: Inductive definition of free variables $FV(\phi)$ and accessed channels $CN(\phi)$ of a formula $\phi \in \text{Fml}(V, \Omega)$, where $\sim \in \{=, \geq, \preceq\}$, and $\Rightarrow \in \{\wedge, \vee, \rightarrow\}$, and $\mathcal{Q} \in \{\forall, \exists\}$.

We start with induction on formula ϕ . By premise, $v \downarrow \phi = \tilde{v} \downarrow \phi$ on $FV(\phi)$. Then we prove that $v \in \llbracket \phi \rrbracket$ implies $\tilde{v} \in \llbracket \phi \rrbracket$. For the converse, swap v and \tilde{v} . We consider $\forall z \neg\varphi$ to be structurally smaller than $\exists z \varphi$ for any formula φ . Note that $v \downarrow D = \tilde{v} \downarrow D$ implies $v \downarrow E = \tilde{v} \downarrow E$ if $E \subseteq D \subseteq \Omega$. We use this fact throughout the proof without further mentioning it.

1. $v \models e_1 \sim e_2$, where $\sim \in \{=, \geq, \preceq\}$, iff $\llbracket e_1 \rrbracket v \sim \llbracket e_2 \rrbracket v$ holds. For $j = 1, 2$, $FV(e_j) \subseteq FV(e_1 \sim e_2)$ and $CN(e_j) \subseteq CN(e_1 \sim e_2)$ such that $v \downarrow e_j = \tilde{v} \downarrow e_j$ on $FV(e_j)$. Hence, $\llbracket e_j \rrbracket v = \llbracket e_j \rrbracket \tilde{v}$ for $j = 1, 2$ by coincidence for terms (Lemma 5), which implies $\llbracket e_1 \rrbracket v \sim \llbracket e_2 \rrbracket \tilde{v}$. Thus, $\tilde{v} \models e_1 \sim e_2$.
2. $v \models \neg\varphi$ implies $v \notin \llbracket \varphi \rrbracket$. By IH, $\tilde{v} \notin \llbracket \varphi \rrbracket$ since $FV(\neg\varphi) = FV(\varphi)$ and $CN(\neg\varphi) = CN(\varphi)$. Hence, $\tilde{v} \models \neg\varphi$.
3. $v \models \varphi \wedge \psi$ iff $v \models \varphi$ and $v \models \psi$. Since $FV(\chi) \subseteq FV(\varphi \wedge \psi)$ and $CN(\chi) \subseteq CN(\varphi \wedge \psi)$ for $\chi \in \{\varphi, \psi\}$, we have $v \downarrow \chi = \tilde{v} \downarrow \chi$ on $FV(\chi)$. Then $\tilde{v} \models \varphi$ and $\tilde{v} \models \psi$ by IH. Finally, $\tilde{v} \models \varphi \wedge \psi$.
4. The cases $\varphi \vee \psi$ and $\varphi \rightarrow \psi$ can be handled analogously to $\varphi \wedge \psi$.
5. $v \models \forall z \varphi$ iff $v_z^a \models \varphi$ for all $a \in \text{type}(z)$. We have $v_z^a \downarrow \varphi = \tilde{v}_z^a \downarrow \varphi$ on $FV(\varphi)$ since $v \downarrow \varphi = \tilde{v} \downarrow \varphi$ on $FV(\forall z \varphi) = FV(\varphi) \setminus \{z\}$ and $CN(\forall z \varphi) = CN(\varphi)$. Hence, by IH, $\tilde{v}_z^a \models \varphi$ for all $a \in \text{type}(z)$, which implies $\tilde{v} \models \forall z \varphi$.
6. $v \models \exists z \varphi$ iff $v \not\models \forall z \neg\varphi$, which implies $\tilde{v} \not\models \forall z \neg\varphi$ by IH because $FV(\forall z \neg\varphi) = FV(\exists z \varphi)$, and $CN(\forall z \neg\varphi) = CN(\exists z \varphi)$, and we considered $\forall z \neg\varphi$ to be structurally smaller than $\exists z \varphi$. Hence, $\tilde{v} \models \exists z \varphi$.
7. $\phi \equiv [\alpha]\psi$, then let $v \models \phi$. To show $\tilde{v} \models \phi$, let $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket$ with $\tilde{w} \neq \perp$. Since $v \downarrow \phi = \tilde{v} \downarrow \phi$ on $FV(\phi) \supseteq FV(\alpha)$, a computation $(v, \tau, w) \in \llbracket \alpha \rrbracket$ with $w \neq \perp$, and $w \downarrow \phi = \tilde{w} \downarrow \phi$ on $FV(\phi) \cup MBV(\alpha)$, and $\tau = \tilde{\tau}$ exists by the simultaneous IH. Now, $w \cdot \tau \models \psi$ because $v \models \phi$. By $FV(\phi) = FV(\alpha) \cup (FV(\psi) \setminus MBV(\alpha))$, we obtain $w \downarrow \phi = \tilde{w} \downarrow \phi$ on $FV(\psi) \subseteq FV(\phi) \cup MBV(\alpha)$, which implies $w \downarrow \psi = \tilde{w} \downarrow \psi$ on $FV(\psi)$ by $CN(\phi) = CN(\psi)$. Furthermore, $(w \cdot \tau) \downarrow \psi = (\tilde{w} \cdot \tilde{\tau}) \downarrow \psi$ because $\tau = \tilde{\tau}$. Thus, IH is applicable on $w \cdot \tau \models \psi$ such that $\tilde{w} \cdot \tilde{\tau} \models \psi$ by IH.
8. $\phi \equiv [\alpha]_{\{A,C\}}\psi$, then let $v \models \phi$. In order to show $\tilde{v} \models \phi$ let $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket$. Since $v \downarrow \phi = \tilde{v} \downarrow \phi$ on $FV(\phi) \supseteq FV(\alpha)$, a computation $(v, \tau, w) \in \llbracket \alpha \rrbracket$ with $w \downarrow \phi = \tilde{w} \downarrow \phi$ on $FV(\phi) \cup MBV(\alpha)$, and $\tau = \tilde{\tau}$, and $(w = \perp \text{ iff } \tilde{w} = \perp)$ exists by the simultaneous IH.

For (commit), assume $\{\tilde{v} \cdot \tau' \mid \tau' \prec \tilde{\tau}\} \models \mathbf{A}$. Then $\{v \cdot \tau' \mid \tau' \prec \tau\} \models \mathbf{A}$ by IH since $\tilde{\tau} = \tau$ and $v \downarrow \phi = \tilde{v} \downarrow \phi$ on $FV(\phi)$ implies $v \downarrow \mathbf{A} = \tilde{v} \downarrow \mathbf{A}$ on $FV(\mathbf{A})$ by $FV(\mathbf{A}) \subseteq FV(\phi)$ and $CN(\mathbf{A}) \subseteq CN(\phi)$. Hence, $v \cdot \tau \models \mathbf{C}$ because $v \models \phi$, which in turn implies $\tilde{v} \cdot \tilde{\tau} \models \mathbf{C}$ by IH because $(v \cdot \tau) \downarrow \mathbf{C} = (\tilde{v} \cdot \tilde{\tau}) \downarrow \mathbf{C}$ on $FV(\mathbf{C})$.

For (post), assume $\tilde{w} \neq \perp$ and $\{\tilde{v} \cdot \tau' \mid \tau' \preceq \tilde{\tau}\} \models \mathbf{A}$. Since $v \downarrow \phi = \tilde{v} \downarrow \phi$ on $FV(\phi) \supseteq FV(\mathbf{A})$ with $CN(\phi) \supseteq CN(\mathbf{A})$, we obtain $\{v \cdot \tau' \mid \tau' \preceq \tau\} \models \mathbf{A}$ by IH. Hence, $w \cdot \tau \models \psi$ by $v \models \phi$. Moreover, $w \downarrow \psi = \tilde{w} \downarrow \psi$ on $FV(\psi)$ because $FV(\psi) \subseteq FV(\phi) \cup MBV(\alpha)$ and $CN(\psi) \subseteq CN(\phi)$. Finally, $\tilde{w} \cdot \tau \models \psi$ by IH.

Now, we proceed with induction on program α . By premise, $v \downarrow C = \tilde{v} \downarrow C$ on $X \supseteq FV(\gamma)$. W.l.o.g. $X \cap V_{\mathcal{T}} = \emptyset$ because $v = w$ and $\tilde{v} = \tilde{w}$ both on $(BV(\alpha) \setminus V_{\mathcal{T}})^{\mathbb{C}}$ by the bound effect property (Lemma 4) such that $\tilde{w} \downarrow C = \tilde{v} \downarrow C = v \downarrow C = w \downarrow C$ on $X \cap V_{\mathcal{T}}$. Hence, $v = v \downarrow C = \tilde{v} \downarrow C = \tilde{v}$ on X . Further, we consider α^n to be structurally smaller than α^* for all programs α and all $n \in \mathbb{N}$, and $x := *$; $\text{ch}(h)!x$ to be smaller than $\text{ch}(h)?x$.

If γ is an atomic non-communicating program, we can handle $(v, \tau, w) \in \llbracket \gamma \rrbracket$ with $w = \perp$ uniformly as follows: Let $(v, \tau, \perp) \in \llbracket \gamma \rrbracket$. Then $\tau = \epsilon$, and we define $\tilde{\tau} = \epsilon$ and $\tilde{w} = \perp$. By totality and prefix-closedness (Proposition 2), $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \gamma \rrbracket$. Further, $w = \tilde{w}$ on $X \cup MBV(\gamma)$ since $\perp = \perp$ on any set of variables. Therefore, we assume $w \neq \perp$ w.l.o.g. in the first four cases below.

1. $(v, \tau, w) \in \llbracket x := \theta \rrbracket$, then $\tau = \epsilon$ and $w = v_x^{\llbracket \theta \rrbracket} v$. We define $\tilde{\tau} = \epsilon$ and $\tilde{w} = \tilde{v}_x^{\llbracket \theta \rrbracket} \tilde{v}$ such that $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket x := \theta \rrbracket$. By coincidence (Lemma 5), $w(x) = \llbracket \theta \rrbracket v = \llbracket \theta \rrbracket \tilde{v} = \tilde{w}(x)$ since $v = \tilde{v}$ on $X \supseteq FV(x := \theta) = FV(\theta)$. Moreover, $w = \tilde{w}$ on $X \setminus \{x\}$ because $v = \tilde{v}$ on X , and $v = w$ and $\tilde{v} = \tilde{w}$ on $\{x\}^{\mathbb{C}}$. Hence, $w = \tilde{w}$ on $X \cup MBV(x := \theta)$ since $MBV(x := \theta) = \{x\}$, and $\tau = \tilde{\tau}$, and $\tilde{w} \neq \perp$.
2. $(v, \tau, w) \in \llbracket x := * \rrbracket$, then $\tau = \epsilon$ and $w = v_x^a$ for some $a \in \mathbb{R}$. We define $\tilde{\tau} = \epsilon$ and $\tilde{w} = \tilde{v}_x^a$. Obviously $v = w$ and $\tilde{v} = \tilde{w}$ on $\{x\}^{\mathbb{C}}$. Thus, $v = \tilde{v}$ on X implies $w = \tilde{w}$ on $X \setminus \{x\}$. Moreover, $w(x) = v_x^a(x) = \tilde{v}_x^a(x) = \tilde{w}(x)$. Overall, $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket$ exists with $w = \tilde{w}$ on $X \cup MBV(x := *)$ since $MBV(x := *) = \{x\}$. Finally, $\tau = \tilde{\tau}$ and $\tilde{w} \neq \perp$.
3. $\gamma \equiv \{x' = \theta \& \chi\}$ and $(v, \tau, w) \in \llbracket \gamma \rrbracket$, then $\tau = \epsilon$ and a solution $\varphi : [0, s] \rightarrow \mathcal{S}$ exists with $v = \varphi(0)$, and $w = \varphi(s)$, and $\varphi(\zeta) \models x' = \theta \wedge \chi$ and $\varphi(\zeta)(\mu) = v(\mu) + \zeta$ for all $\zeta \in [0, s]$, where $\varphi(\zeta)(x') = \frac{d\varphi(t)(x)}{dt}(\zeta)$, and $\varphi(\zeta)(y) = v(y)$ for $y \notin \{x, \mu\}$. We define a solution $\tilde{\varphi} : [0, s] \rightarrow \mathcal{S}$ of equal duration s for $\zeta \in [0, s]$, by $\tilde{\varphi}(\zeta) = \varphi(\zeta)$ on $\{x, \mu\}$ and if $y \notin \{x, \mu\}$ by $\tilde{\varphi}(\zeta)(y) = \tilde{v}(y)$. Since $\varphi(\zeta)(x) = \tilde{\varphi}(\zeta)(x)$ for all $\zeta \in [0, s]$, we have $\varphi(\zeta)(x') = \tilde{\varphi}(\zeta)(x')$. Moreover, $\tilde{\varphi}(\zeta) = \varphi(\zeta)$ on $\{x, \mu\}$ and $\varphi(\zeta) = v = \tilde{v} = \tilde{\varphi}(\zeta)$ on $X \setminus \{x, \mu\}$ imply $\varphi(\zeta) = \tilde{\varphi}(\zeta)$ on $X \cup MBV(\gamma)$ because $MBV(\gamma) = \{x, \mu\}$. Since $X \supseteq FV(\gamma) \supseteq FV(\theta) \cup FV(\chi)$ by premise, $\llbracket \theta \rrbracket \varphi(\zeta) = \llbracket \theta \rrbracket \tilde{\varphi}(\zeta)$ by coincidence for terms (Lemma 5). Moreover, $\varphi(\zeta) \models \chi$ iff $\tilde{\varphi}(\zeta) \models \chi$ by the simultaneous IH. Thus, $\tilde{\varphi}(\zeta) \models x' = \theta \wedge \chi$ for all $\zeta \in [0, s]$. Moreover, $\tilde{\varphi}(\zeta)(y) = \tilde{v}(y)$ for $y \notin \{x, \mu\}$ by definition of $\tilde{\varphi}$. If we define $\tilde{v} = \tilde{\varphi}(0)$, and $\tilde{\tau} = \epsilon$, and $\tilde{w} = \tilde{\varphi}(s)$, then $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket$. Finally, $\tilde{w} = \tilde{\varphi}(s) = \varphi(s) = w$ on $X \cup MBV(\gamma)$, and $\tau = \tilde{\tau}$, and $\tilde{w} \neq \perp$.

4. $(v, \tau, w) \in \llbracket ?\chi \rrbracket$, then $\tau = \epsilon$, and $w = v$, and $v \models \chi$. We define $\tilde{\tau} = \epsilon$ and $\tilde{w} = \tilde{v}$. Since $v \models \chi$ and $v = \tilde{v}$ on $X \supseteq FV(?\chi) = FV(\chi)$, we obtain $\tilde{v} \models \chi$ by the simultaneous IH. Hence, $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket ?\chi \rrbracket$. Moreover, $w = v = \tilde{v} = \tilde{w}$ on $X \cup MBV(?\chi)$ because $MBV(?\chi) = \emptyset$. Finally, $\tau = \tilde{\tau}$ and $\tilde{w} \neq \perp$.
5. $\gamma \equiv \text{ch}(h)! \theta$ and $(v, \tau, w) \in \llbracket \gamma \rrbracket$, then $(\tau, w) \preceq (\langle h, \text{ch}, \llbracket \theta \rrbracket v, v(\mu) \rangle, v)$. We define $\tilde{\tau} = \tau$, and $\tilde{w} = \perp$ if $w = \perp$ and $\tilde{w} = \tilde{v}$ otherwise. Since $v = \tilde{v}$ on $X \supseteq FV(\gamma) = \{h, \mu\} \cup FV(\theta)$, we have $\llbracket \theta \rrbracket v = \llbracket \theta \rrbracket \tilde{v}$ and $v(\mu) = \tilde{v}(\mu)$. Hence, $(\tilde{\tau}, \tilde{w}) \preceq (\langle h, \text{ch}, \llbracket \theta \rrbracket \tilde{v}, \tilde{v}(\mu) \rangle, \tilde{v})$, which implies $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \gamma \rrbracket$. Finally, if $w \neq \perp$, then $w = \tilde{w}$ on $X \cup MBV(\gamma) = X \cup \{h\}$ because $w = v = \tilde{v} = \tilde{w}$ on X since $v = \tilde{v}$ on $X \subseteq FV(\gamma) \supseteq \{h\}$. Moreover, $\tau = \tilde{\tau}$ and $(w = \perp \text{ iff } \tilde{w} = \perp)$.
6. $(v, \tau, w) \in \llbracket \text{ch}(h)?x \rrbracket$, then $(v, \tau, w) \in \llbracket \alpha \rrbracket$, where $\alpha \equiv x := *; \text{ch}(h)!x$. Since $FV(\text{ch}(h)?x) = FV(\alpha)$, we have $v = \tilde{v}$ on $X \supseteq FV(\alpha)$. Hence, by IH, $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket$ exists with $\tau = \tilde{\tau}$ and $w = \tilde{w}$ on $X \cup MBV()$ and $(w = \perp \text{ iff } \tilde{w} = \perp)$ because we considered α to be structurally smaller than $\text{ch}(h)?x$. Finally, $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \text{ch}(h)?x \rrbracket$ and observe that $MBV(\text{ch}(h)?x) = MBV(\alpha)$.
7. $(v, \tau, w) \in \llbracket \alpha_1 \cup \alpha_2 \rrbracket$, then $(v, \tau, w) \in \llbracket \alpha_j \rrbracket$ for some $j \in \{1, 2\}$. Since $X \supseteq FV(\alpha_1 \cup \alpha_2) \supseteq FV(\alpha_j)$, we have $v = \tilde{v}$ on $X \supseteq FV(\alpha_j)$. Thus, $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha_j \rrbracket \subseteq \llbracket \alpha_1 \cup \alpha_2 \rrbracket$ exists by IH such that $w = \tilde{w}$ on $X \cup MBV(\alpha_j)$, and $\tau = \tilde{\tau}$, and $(w = \perp \text{ iff } \tilde{w} = \perp)$. Finally, $w = \tilde{w}$ on $X \cup MBV(\alpha_1 \cup \alpha_2)$ because $MBV(\alpha_1 \cup \alpha_2) = MBV(\alpha_1) \cap MBV(\alpha_2)$.
8. $(v, \tau, w) \in \llbracket \alpha; \beta \rrbracket$, then $(v, \tau, w) \in \llbracket \alpha \rrbracket_{\perp}$ or $(v, \tau, w) \in \llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket$.
 If $(v, \tau, w) \in \llbracket \alpha \rrbracket_{\perp} \subseteq \llbracket \alpha \rrbracket$, a computation $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket$ exists by IH such that $\tau = \tilde{\tau}$ and $\tilde{w} = \perp$. Since $w = \tilde{w} = \perp$, we have $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket_{\perp} \subseteq \llbracket \alpha; \beta \rrbracket$ with $w = \tilde{w}$ on $X \cup MBV(\alpha; \beta)$, and $\tau = \tilde{\tau}$, and $(w' = \perp \text{ iff } \tilde{w} = \perp)$.
 If $(v, \tau, w) \in \llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket$, computations $(v, \tau_1, u) \in \llbracket \alpha \rrbracket$ and $(u, \tau_2, w) \in \llbracket \beta \rrbracket$ with $\tau = \tau_1 \cdot \tau_2$ exist. Since $X \supseteq FV(\alpha; \beta) \supseteq FV(\alpha)$, by IH, $(\tilde{v}, \tilde{\tau}_1, \tilde{u}) \in \llbracket \alpha \rrbracket$ exists with $u = \tilde{u}$ on $X \cup MBV(\alpha)$, and $\tau_1 = \tilde{\tau}_1$, and $\tilde{u} \neq \perp$ because $u \neq \perp$. Observe that $X \cup MBV(\alpha) \supseteq FV(\alpha; \beta) \cup MBV(\alpha) = FV(\alpha) \cup (FV(\beta) \setminus MBV(\alpha)) \cup MBV(\alpha) \supseteq FV(\beta)$. Therefore, $(\tilde{u}, \tilde{\tau}_2, \tilde{w}) \in \llbracket \beta \rrbracket$ exists by IH again with $w = \tilde{w}$ on $X \cup MBV(\alpha) \cup MBV(\beta) = X \cup MBV(\alpha; \beta)$, $\tau_2 = \tilde{\tau}_2$, and $(w = \perp \text{ iff } \tilde{w} = \perp)$. Reassembling gives us $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket \subseteq \llbracket \alpha; \beta \rrbracket$ with $\tilde{\tau} = \tilde{\tau}_1 \cdot \tilde{\tau}_2$. Finally, $w = \tilde{w}$ on $X \cup MBV(\alpha; \beta)$, and $\tau = \tilde{\tau}$, and $(w = \perp \text{ iff } \tilde{w} = \perp)$.
9. $(v, \tau, w) \in \llbracket \alpha^* \rrbracket$, then $(v, \tau, w) \in \llbracket \alpha^n \rrbracket$ for some $n \in \mathbb{N}$. By induction on n , we show that $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha^n \rrbracket$ exists with $w = \tilde{w}$ on X , and $\tau = \tilde{\tau}$, and $(w = \perp \text{ iff } \tilde{w} = \perp)$. Finally, we conclude by $\llbracket \alpha^n \rrbracket \subseteq \llbracket \alpha^* \rrbracket$ and $MBV(\alpha^*) = \emptyset$.
 - (a) $n = 0$: $(v, \tau, w) \in \llbracket \alpha^0 \rrbracket = \llbracket ?\top \rrbracket$, then $\tau = \epsilon$, and $w = \perp$ or $w = v$. We define $\tilde{\tau} = \epsilon$, and $\tilde{w} = \perp$ if $w = \perp$ and $\tilde{w} = \tilde{v}$ otherwise. Hence, $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha^0 \rrbracket$ and $w = \tilde{w}$ on X because if $w \neq \perp$, then $v = \tilde{v}$ on X implies $w = v = \tilde{v} = \tilde{w}$ on X . Moreover, $\tau = \tilde{\tau}$, and $(w = \perp \text{ iff } \tilde{w} = \perp)$.
 - (b) $n > 0$: $(v, \tau, w) \in \llbracket \alpha^n \rrbracket$, then $(v, \tau, w) \in \llbracket \alpha; \alpha^{n-1} \rrbracket = \llbracket \alpha \rrbracket_{\perp} \cup (\llbracket \alpha \rrbracket \triangleright \llbracket \alpha^{n-1} \rrbracket)$.
 If $(v, \tau, w) \in \llbracket \alpha \rrbracket_{\perp} \subseteq \llbracket \alpha \rrbracket$, by IH, $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket$ exists with $\tau = \tilde{\tau}$ and $\tilde{w} = \perp$. That is, $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket_{\perp} \subseteq \llbracket \alpha^n \rrbracket$ with $w = \tilde{w}$ on X since $\perp = \perp$ on any set of variables, $\tau = \tilde{\tau}$, and $(w = \perp \text{ iff } \tilde{w} = \perp)$.
 If $(v, \tau, w) \in \llbracket \alpha \rrbracket \triangleright \llbracket \alpha^{n-1} \rrbracket$, computations $(v, \tau_1, u) \in \llbracket \alpha \rrbracket$ and $(u, \tau_2, w) \in \llbracket \alpha^{n-1} \rrbracket$ exist with $\tau = \tau_1 \cdot \tau_2$. Since $X \supseteq FV(\alpha^*) = FV(\alpha)$, by IH,

$(\tilde{v}, \tilde{\tau}_1, \tilde{u}) \in \llbracket \alpha \rrbracket$ exists with $u = \tilde{u}$ on X , and $\tau_1 = \tilde{\tau}_1$, and $\tilde{u} \neq \perp$. Since $X \supseteq FV(\alpha^*) = FV(\alpha^{n-1})$ and α^{n-1} is structurally smaller than α^* , by IH again, $(\tilde{u}, \tilde{\tau}_2, \tilde{w}) \in \llbracket \alpha^{n-1} \rrbracket$ with $w = \tilde{w}$ on X , and $\tau_2 = \tilde{\tau}_2$, and ($w = \perp$ iff $\tilde{w} = \perp$). Reassembling gives us $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha \rrbracket \triangleright \llbracket \alpha^{n-1} \rrbracket \subseteq \llbracket \alpha^n \rrbracket$ with $\tilde{\tau} = \tilde{\tau}_1 \cdot \tilde{\tau}_2$. Finally, $w = \tilde{w}$ on X , and $\tau = \tilde{\tau}$, and ($w = \perp$ iff $\tilde{w} = \perp$).

10. $(v, \tau, w) \in \llbracket \alpha_1 \parallel \alpha_2 \rrbracket$, then $(v, \tau \downarrow \alpha_j, w_{\alpha_j}) \in \llbracket \alpha \rrbracket$ exists for $j = 1, 2$ such that $w_{\alpha_1}(\mu) = w_{\alpha_2}(\mu)$, and $\tau = \tau \downarrow (\alpha_1 \parallel \alpha_2)$ and $w = w_{\alpha_1} \oplus w_{\alpha_2}$. Since $X \supseteq FV(\alpha_1 \parallel \alpha_2) \supseteq FV(\alpha_j)$, by IH, $(\tilde{v}, \tilde{\tau}_{\alpha_j}, \tilde{w}_{\alpha_j}) \in \llbracket \alpha_j \rrbracket$ exists with $w_{\alpha_j} = \tilde{w}_{\alpha_j}$ on $X \cup MBV(\alpha_j)$, and $\tau \downarrow \alpha_j = \tilde{\tau}_{\alpha_j}$, and ($w_{\alpha_j} = \perp$ iff $\tilde{w}_{\alpha_j} = \perp$). We define $\tilde{\tau} = \tau$. Thus, $\tilde{\tau} \downarrow \alpha_j = \tilde{\tau}_{\alpha_j}$, and $\tilde{\tau} = \tilde{\tau} \downarrow (\alpha_1 \parallel \alpha_2)$. Moreover, $\tilde{w}_{\alpha_1}(\mu) = \tilde{w}_{\alpha_2}(\mu)$ either because $\tilde{w}_{\alpha_1} = \tilde{v} = \tilde{w}_{\alpha_2}$ on μ by Lemma 4 if μ is not bound in both programs, or because $\mu \in X$ since $\mu \in BV(\alpha_1 \parallel \alpha_2)$ implies $\mu \in FV(\alpha_1 \parallel \alpha_2)$ such that $\tilde{w}_{\alpha_1} = w_{\alpha_1} = w_{\alpha_2} = \tilde{w}_{\alpha_2}$ on μ . In summary, $(\tilde{v}, \tilde{\tau}, \tilde{w}) \in \llbracket \alpha_1 \parallel \alpha_2 \rrbracket$, where $\tilde{w} = \tilde{w}_{\alpha_1} \oplus \tilde{w}_{\alpha_2}$, such that $\tau = \tilde{\tau}$, and ($w = \perp$ iff $\tilde{w} = \perp$).

Finally, we show $w = \tilde{w}$ on $X \cup MBV(\alpha_1 \parallel \alpha_2)$. Let $z \in X \cup MBV(\alpha_1 \parallel \alpha_2) = X \cup MBV(\alpha_1) \cup MBV(\alpha_2)$. If $z \in BV(\alpha_1)$, then $z \in X \cup MBV(\alpha_1) \cup \{\mu\} \cup V_{\mathcal{T}}$ because $V(\alpha_2) \cap BV(\alpha_1) \subseteq \{\mu\} \cup V_{\mathcal{T}}$ (well-formedness) and $MBV(\alpha_2) \subseteq V(\alpha_2)$. Moreover, $BV(\alpha_1) \cap (\{\mu\} \cup V_{\mathcal{T}}) \subseteq MBV(\alpha_1)$. Now, $w(z) = w_{\alpha_1}(z) = \tilde{w}_{\alpha_1}(z) = \tilde{w}(z)$ since $w_{\alpha_1} = \tilde{w}_{\alpha_1}$ on $X \cup MBV(\alpha_1)$, and moreover, $w = w_{\alpha_1}$ on $BV(\alpha_1)$ and $\tilde{w} = \tilde{w}_{\alpha_1}$ on $BV(\alpha_1)$ by the definition of \oplus in Section 2.2. If $z \notin BV(\alpha_1)$, then $z \in X \cup MBV(\alpha_2)$. We conclude by $w(z) = w_{\alpha_1}(z) = \tilde{w}_{\alpha_2}(z) = \tilde{w}(z)$ because $w = w_{\alpha_2}$ on $BV(\alpha_1)^c$ and $\tilde{w} = \tilde{w}_{\alpha_2}$ on $BV(\alpha_1)^c$ by the definition of \oplus again, and $w_{\alpha_2} = \tilde{w}_{\alpha_2}$ on $X \cup MBV(\alpha_2)$. \square

C Details of the Calculus

Reasoning on trace-terms is by simple algebraic laws (see Fig. 15). Their soundness should be clear from the semantics of terms (see Def. 9).

$$\begin{array}{ll}
 \cdot \downarrow & (te_1 \cdot te_2) \downarrow C = te_1 \downarrow C \cdot te_2 \downarrow C & \cdot A & (te_1 \cdot te_2) \cdot te_3 = te_1 \cdot (te_2 \cdot te_3) \\
 \downarrow \in & \langle \text{ch}, \theta_1, \theta_2 \rangle \downarrow C = \langle \text{ch}, \theta_1, \theta_2 \rangle \quad (\text{ch} \in C) & \cdot N & te \cdot \epsilon = te = \epsilon \cdot te \\
 \downarrow \cap & (te \downarrow C') \downarrow C = te \downarrow (C' \cap C) & \epsilon \downarrow & \epsilon \downarrow C = \epsilon \\
 \text{val} = & |te| = ie \rightarrow \text{val}((te \cdot te_0)[ie]) = \theta_1 & \downarrow \notin & \langle \text{ch}, \theta_1, \theta_2 \rangle \downarrow C = \epsilon \quad (\text{ch} \notin C) \\
 \text{time} = & |te| = ie \rightarrow \text{time}((te \cdot te_0)[ie]) = \theta_2 & |\cdot| \geq 0 & |te| \geq 0 \\
 \text{chan} = & |te| = ie \rightarrow \text{chan}((te \cdot te_0)[ie]) = \text{ch} & |\cdot| + 1 & |te \cdot te_0| = |te| + 1 \\
 \\
 \text{val} > & |te| > ie \rightarrow \text{val}((te \cdot te_0)[ie]) = \text{val}(te[ie]) \\
 \text{time} > & |te| > ie \rightarrow \text{time}((te \cdot te_0)[ie]) = \text{time}(te[ie]) \\
 \text{chan} > & |te| > ie \rightarrow \text{chan}((te \cdot te_0)[ie]) = \text{chan}(te[ie])
 \end{array}$$

Fig. 15: Algebra of traces [42], where $te_0 \equiv \langle \text{ch}, \theta_1, \theta_2 \rangle$.

In preparation for the soundness proof (see Theorem 1), Lemma 8 shows that A and C do not depend on the state of α if $[\alpha]_{\{A,C\}}\psi$ is well-formed. Moreover, Lemma 9 exploits Def. 7 showing that a program β not interfering with $[\alpha]_{\{A,C\}}\psi$ has no influence on the validity of A , C , and ψ in the parallel composition $[\alpha \parallel \beta]_{\{A,C\}}\psi$.

Lemma 8. *Let $[\alpha]_{\{A,C\}}\psi$ be well-formed and $(v, \tau, w) \in \llbracket \alpha \rrbracket$ with $w \neq \perp$. Then $v = w$ on $FV(A) \cup FV(C)$. Moreover, $v \cdot \tilde{\tau} \models \chi$ iff $w \cdot \tilde{\tau} \models \chi$ for $\chi \in \{A, C\}$ and arbitrary $\tilde{\tau} \in \mathcal{T}_{rec}$.*

Proof. By the bound effect property (Lemma 4), $v = w$ on $BV(\alpha)^{\mathbb{G}} \cup V_{\mathcal{T}}$. Since $[\alpha]_{\{A,C\}}\psi$ is well-formed, we have $(FV(A) \cup FV(C)) \cap BV(\alpha) \subseteq V_{\mathcal{T}}$, which implies $v = w$ on $FV(A) \cup FV(C)$. By coincidence (Lemma 6), $v \cdot \tilde{\tau} \models \chi$ iff $w \cdot \tilde{\tau} \models \chi$. \square

Lemma 9 (Noninterference prevents invalidation). *Let $\beta \in \text{CHP}(V, \Omega)$ be a program, which does not interfere with $[\alpha]_{\{A,C\}}\psi$ (Def. 7). Moreover, let $(v, \tau, w) \in \llbracket \alpha \parallel \beta \rrbracket$, i.e., among others $(v, \tau \downarrow \alpha, w_{\alpha}) \in \llbracket \alpha \rrbracket$ and $(v, \tau \downarrow \beta, w_{\beta}) \in \llbracket \beta \rrbracket$ with $w = w_{\alpha} \oplus w_{\beta}$. Then for $\chi \in \{A, C\}$, the following holds:*

1. $v \cdot (\tau \downarrow \alpha) \models \chi$ iff $v \cdot \tau \models \chi$
2. $w \neq \perp$ implies $(w_{\alpha} \cdot (\tau \downarrow \alpha)) \models \psi$ iff $w \cdot \tau \models \psi$

Proof. First, we show $(\tau \downarrow \alpha) \downarrow \lambda = \tau \downarrow \lambda$ for $\lambda \in \{A, C, \psi\}$, i.e., that λ only depends on $\tau \downarrow \alpha$. This holds if only a communication event $\rho \in \{\langle \text{ch}, a, s \rangle, \langle h, \text{ch}, a, s \rangle\}$ in τ , which is not removed by $\downarrow \lambda$ is also not removed by $\downarrow \alpha$. So let $\rho \downarrow \lambda = \rho$. Then $\text{ch} \in CN(\lambda)$. If $\text{ch} \notin CN(\beta)$, then $\text{ch} \in CN(\alpha)$ because $\text{ch} \in CN(\alpha) \cup CN(\beta)$ as ρ is emitted by $\alpha \parallel \beta$. Otherwise, if $\text{ch} \in CN(\beta)$, then $\text{ch} \in CN(\alpha)$ by noninterference (Def. 7). Hence, $\text{ch} \in CN(\alpha)$ such that ρ is not removed by $\downarrow \alpha$.

Since $(\tau \downarrow \alpha) \downarrow \chi = \tau \downarrow \chi$, we have

$$(v \cdot (\tau \downarrow \alpha)) \downarrow \chi = (v \downarrow \chi) \cdot ((\tau \downarrow \alpha) \downarrow \chi) = (v \downarrow \chi) \cdot (\tau \downarrow \chi) = (v \cdot \tau) \downarrow \chi \quad (5)$$

on $V \supseteq FV(\chi)$. Now, item 1 follows by coincidence (Lemma 6).

For item 2, assume $w \neq \perp$. Then $w_{\alpha} \neq \perp$ and $w_{\beta} \neq \perp$ by the definition of \oplus in Section 2.2. First, observe that $w_{\alpha} = w$ on $BV(\alpha)$ by the definition of \oplus , thus $w_{\alpha} = w$ on $BV(\alpha) \cap BV(\beta)^{\mathbb{G}}$. Second, $w_{\alpha} = v$ on $BV(\alpha)^{\mathbb{G}} \cup V_{\mathcal{T}}$ by the bound effect property (Lemma 4), $v = w_{\beta}$ on $BV(\beta)^{\mathbb{G}} \cup V_{\mathcal{T}}$ also by the bound effect property, and $w_{\beta} = w$ on $BV(\alpha)^{\mathbb{G}}$ by the definition of \oplus . Chaining the equalities of the last sentence gives us $w_{\alpha} = w$ on

$$(BV(\alpha)^{\mathbb{G}} \cup V_{\mathcal{T}}) \cap (BV(\beta)^{\mathbb{G}} \cup V_{\mathcal{T}}) \cap BV(\alpha)^{\mathbb{G}} \supseteq BV(\alpha)^{\mathbb{G}} \cap BV(\beta)^{\mathbb{G}}.$$

Third, $w_{\alpha} = v = w_{\beta}$ on $V_{\mathcal{T}}$ by the bound effect property such that $w_{\alpha} = w_{\alpha} \oplus w_{\beta} = w$ on $V_{\mathcal{T}}$. Summarizing, where w_{α} equals w , we obtain $w_{\alpha} = w$ on

$$(BV(\alpha) \cap BV(\beta)^{\mathbb{G}}) \cup (BV(\alpha)^{\mathbb{G}} \cap BV(\beta)^{\mathbb{G}}) \cup V_{\mathcal{T}} = BV(\beta)^{\mathbb{G}} \cup V_{\mathcal{T}}.$$

Thus, $w_{\alpha} = w$ on $FV(\psi)$ because β does not interfere with $[\alpha]_{\{A,C\}}\psi$ (Def. 7), which implies $FV(\psi) \subseteq BV(\beta)^{\mathbb{G}} \cup V_{\mathcal{T}}$. Therefore, $(w_{\alpha} \cdot (\tau \downarrow \alpha)) \downarrow \chi = (w \cdot \tau) \downarrow \chi$ on $FV(\psi)$ since $(w_{\alpha} \cdot (\tau \downarrow \alpha)) \downarrow \chi = (w_{\alpha} \cdot \tau) \downarrow \chi$ holds analogously to equation (5). Finally, item 2 holds by coincidence. \square

Proposition 3. *The parallel operator \parallel is associative and commutative.*

Proof (Proposition 3). We prove that the parallel operator \parallel is associative and commutative:

1. Let $\alpha\beta$ be short for $\alpha \parallel \beta$. Now, $(v, \tau, w) \in \llbracket \alpha \parallel \beta\gamma \rrbracket$ iff $(v, \tau \downarrow \pi, w_\pi) \in \llbracket \pi \rrbracket$ for $\pi \in \{\alpha, \beta\gamma\}$, and $w_\alpha(\mu) = w_{\beta\gamma}(\mu)$, and $\tau \downarrow (\alpha \parallel \beta\gamma) = \tau$, and $w = w_\alpha \oplus w_{\beta\gamma}$. Moreover, $(v, \tau \downarrow \beta\gamma, w_{\beta\gamma}) \in \llbracket \beta\gamma \rrbracket$ iff $(v, \tau \downarrow \pi, w_\pi) \in \llbracket \pi \rrbracket$ since $(\tau \downarrow \beta\gamma) \downarrow \pi = \tau \downarrow \pi$ for $\pi \in \{\beta, \gamma\}$, and $(\tau \downarrow \beta\gamma) \downarrow \beta\gamma = \tau \downarrow \beta\gamma$, and $w_\beta(\mu) = w_\gamma(\mu)$, and $w_{\beta\gamma} = w_\beta \oplus w_\gamma$.
Likewise, $(v, \tau, w) \in \llbracket \alpha\beta \parallel \gamma \rrbracket$ iff $(v, \tau \downarrow \pi, w_\pi) \in \llbracket \pi \rrbracket$ for $\pi \in \{\alpha, \beta, \gamma\}$, and $w_{\alpha\beta}(\mu) = w_\gamma(\mu)$, and $w_\alpha(\mu) = w_\beta(\mu)$, and $\tau \downarrow (\alpha\beta \parallel \gamma) = \tau$, and $(\tau \downarrow \alpha\beta) \downarrow \alpha\beta = \tau \downarrow \alpha\beta$, and $w = w_{\alpha\beta} \oplus w_\gamma$, and $w_{\alpha\beta} = w_\alpha \oplus w_\beta$.
Both decompositions can be reassembled into the other one because $\tau \downarrow C$ is chronological for any C as τ is chronological. Moreover, all $w_\pi(\mu)$ with $\pi \in \{\alpha, \beta, \gamma\}$ equate since $w_{\beta\gamma}(\mu) = w_\beta(\mu)$ and $w_{\alpha\beta}(\mu) = w_\alpha(\mu)$. Finally, $w_\alpha \oplus (w_\beta \oplus w_\gamma) = (w_\alpha \oplus w_\beta) \oplus w_\gamma$.
If $w_\pi = \perp$ for some $\pi \in \{\alpha, \beta, \gamma\}$, then both sides equal \perp . Otherwise, if $w_\pi \neq \perp$ for all $\pi \in \{\alpha, \beta, \gamma\}$, the equation holds because each variable is written by at most one of α , β , or γ .
2. Let $(v, \tau, w) \in \llbracket \alpha \parallel \beta \rrbracket$. Then $(v, \tau \downarrow \alpha, w_\alpha) \in \llbracket \alpha \rrbracket$, and $(v, \tau \downarrow \beta, w_\beta) \in \llbracket \beta \rrbracket$, and $\tau \downarrow (\alpha \parallel \beta) = \tau$, and $w_\alpha(\mu) = w_\beta(\mu)$, and $w = w_\alpha \oplus w_\beta$. First, $\tau \downarrow (\alpha \parallel \beta) = \tau \downarrow (\beta \parallel \alpha)$ since $CN(\alpha \parallel \beta) = CN(\alpha) \cup CN(\beta)$. By the bound effect property (Lemma 4), $w_\alpha \oplus w_\beta = v = w_\beta \oplus w_\alpha$ on $(BV(\alpha) \cup BV(\beta))^{\mathbb{G}}$. Moreover, $w_\alpha \oplus w_\beta = w_\alpha = w_\beta \oplus w_\alpha$ on $BV(\alpha)$ since $BV(\alpha) \subseteq BV(\beta)^{\mathbb{G}}$ because α and β do not share state. Accordingly, $w_\alpha \oplus w_\beta = w_\beta \oplus w_\alpha$ on $BV(\beta)$ such that overall $w_\alpha \oplus w_\beta = w_\beta \oplus w_\alpha$. Finally, $(v, \tau, w) \in \llbracket \beta \parallel \alpha \rrbracket$. \square

Proof (Theorem 1). We prove soundness of the novel ac-axioms and rules. Since $d\mathcal{L}_{\text{CHP}}$ is a conservative extension of $d\mathcal{L}$ (Proposition 1), we can soundly use the $d\mathcal{L}$ proof calculus for reasoning about $d\mathcal{L}_{\text{CHP}}$ formulas. Hence, we point to the literature for soundness of the axioms and rules adopted from $d\mathcal{L}$ [30, 33, 34].

$[\epsilon]_{\text{AC}}$: Let $v \vDash [\alpha]_{\{A, C\}} \psi$. Then $v \vDash C$ by axiom $\mathbf{W}[\cdot]_{\text{AC}}$. Now, let $(v, \tau, w) \in \llbracket \alpha \rrbracket$, and assume $w \neq \perp$ and $v \vDash A$. Then $\tau = \epsilon$ because $CN(\alpha) = \emptyset$. Therefore, the precondition $\{v \cdot \tau' \mid \tau' \preceq \tau\} \vDash A$ of (post) is fulfilled, and we conclude $w \vDash \psi$. Conversely, let $v \vDash C \wedge (A \rightarrow [\alpha] \psi)$ and $(v, \tau, w) \in \llbracket \alpha \rrbracket$. Condition (commit) holds since $\tau = \epsilon$ and $v \vDash C$ by precondition. For (post), assume $w \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau\} \vDash A$, which contains $v \vDash A$. Then $v \vDash A \rightarrow [\alpha] \psi$ implies $w \vDash \psi$.

$\mathbf{W}[\cdot]_{\text{AC}}$: Let $v \vDash [\alpha]_{\{A, C\}} \psi$. Then $v \vDash C$ by (commit) since $(v, \epsilon, \perp) \in \llbracket \alpha \rrbracket$ by totality (Proposition 2) and $\{v \cdot \tau' \mid \tau' \prec \epsilon\} \vDash A$ trivially holds as $\{v \cdot \tau' \mid \tau' \prec \epsilon\} = \emptyset$. For $[\alpha]_{\{A, C\}}(C \wedge (A \rightarrow \psi))$, (commit) holds by assumption. For (post), let $(v, \tau, w) \in \llbracket \alpha \rrbracket$, and assume $w \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau\} \vDash A$. By (commit), $v \cdot \tau \vDash C$ such that $w \cdot \tau \vDash C$ by Lemma 8. From (post), we obtain $w \cdot \tau \vDash \psi$, which implies $w \cdot \tau \vDash A \rightarrow \psi$. The converse direction derives by rule $\mathbf{M}[\cdot]_{\text{AC}}$.

$\llbracket K \rrbracket$: Let $v \models \text{Cl}_\forall K$, where $K \equiv (A \wedge C_1 \rightarrow A_2) \wedge (A \wedge C_2 \rightarrow A_1)$ and $\text{Cl}_\forall K$ denotes the universal closure of K . Moreover, let $v \models \Phi$ for $\Phi \equiv [\alpha]_{\{A_1 \wedge A_2, C_1 \wedge C_2\}} \psi$ and $(v, \tau, w) \in \llbracket \alpha \rrbracket$. By induction on the length of τ , we simultaneously prove $v \models [\alpha]_{\{A, C_1 \wedge C_2\}} \psi$, and $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A$ implies $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A_1 \wedge A_2$:

1. $|\tau| = 0$, then $v \models \Phi$ implies $v \models C_1 \wedge C_2$ by axiom $\text{W}[\mathcal{A}\mathcal{C}]$. Hence, (commit) holds since $\tau = \epsilon$. For (post), assume $w \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau\} \models A$. Then $\{v \cdot \tau' \mid \tau' \preceq \tau\} \models A_1 \wedge A_2$ since $v \models C_1 \wedge C_2$ holds, K is valid, and $\tau = \epsilon$. Thus, $w \cdot \tau \models \psi$ by assumption. Note that $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A_1 \wedge A_2$ is trivially fulfilled since $\{v \cdot \tau' \mid \tau' \prec \tau\} = \emptyset$.
2. $|\tau| > 0$, then $\tau = \tau_0 \cdot \rho$ with $|\rho| = 1$. For (commit), assume $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A$. Then $\{v \cdot \tau' \mid \tau' \preceq \tau_0\} \models A$, which implies $\{v \cdot \tau' \mid \tau' \prec \tau_0\} \models A_1 \wedge A_2$ by IH. Since $(v, \tau_0, \perp) \in \llbracket \alpha \rrbracket$ by prefix-closedness (Proposition 2), we obtain $v \cdot \tau_0 \models C_1 \wedge C_2$ by $v \models \Phi$. Hence, $v \cdot \tau_0 \models A_1 \wedge A_2$ as K is valid and $v \cdot \tau_0 \models A$ such that $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A_1 \wedge A_2$. Finally, $v \cdot \tau \models C_1 \wedge C_2$ by $v \models \Phi$ again. For (post), assume $w \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau\} \models A$. Then $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A$, which implies $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A_1 \wedge A_2$ and $v \cdot \tau \models C_1 \wedge C_2$ as in case (commit). By validity of K and $v \cdot \tau \models A$, we obtain $v \cdot \tau \models A_1 \wedge A_2$. In summary, $\{v \cdot \tau' \mid \tau' \preceq \tau\} \models A_1 \wedge A_2$, which implies $w \cdot \tau \models \psi$ by $v \models \Phi$.

$[\text{;}]_{\mathcal{A}\mathcal{C}}$ Let $v \models [\alpha; \beta]_{\{A, C\}} \psi$. To show $v \models [\alpha]_{\{A, C\}} [\beta]_{\{A, C\}} \psi$, let $(v, \tau_1, u) \in \llbracket \alpha \rrbracket$. For (commit), assume $\{v \cdot \tau' \mid \tau' \prec \tau_1\} \models A$ and observe that $(v, \tau_1, \perp) \in \llbracket \alpha \rrbracket_\perp \subseteq \llbracket \alpha; \beta \rrbracket$. Then $v \cdot \tau_1 \models C$ by assumption. For (post), assume $u \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau_1\} \models A$. To show $u \cdot \tau_1 \models [\beta]_{\{A, C\}} \psi$, let $(u, \tau_2, w) \in \llbracket \beta \rrbracket$.

1. For (commit), assume $\{u \cdot \tau_1 \cdot \tau' \mid \tau' \prec \tau_2\} \models A$. Then $\{v \cdot \tau_1 \cdot \tau' \mid \tau' \prec \tau_2\} \models A$ by Lemma 8, which implies $\{v \cdot \tau' \mid \tau' \prec \tau_1 \cdot \tau_2\} \models A$ by assumption $\{v \cdot \tau' \mid \tau' \preceq \tau_1\} \models A$. Then $v \cdot \tau_1 \cdot \tau_2 \models C$ because $(v, \tau_1 \cdot \tau_2, w) \in \llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket \subseteq \llbracket \alpha; \beta \rrbracket$. Finally, $u \cdot \tau_1 \cdot \tau_2 \models C$, using Lemma 8 again.
2. For (post), assume $w \neq \perp$ and $\{u \cdot \tau_1 \cdot \tau' \mid \tau' \preceq \tau_2\} \models A$. Then $\{v \cdot \tau' \mid \tau' \preceq \tau_1 \cdot \tau_2\} \models A$ by Lemma 8. Since $(v, \tau_1 \cdot \tau_2, w) \in \llbracket \alpha; \beta \rrbracket$, we obtain $w \cdot \tau_1 \cdot \tau_2 \models \psi$ by assumption.

Conversely, let $v \models [\alpha]_{\{A, C\}} [\beta]_{\{A, C\}} \psi$ and $(v, \tau, w) \in \llbracket \alpha; \beta \rrbracket$. If $(v, \tau, w) \in \llbracket \alpha \rrbracket_\perp$, (commit) holds by the assumption. Since $w = \perp$, (post) holds trivially. Otherwise, if $(v, \tau, w) \in \llbracket \alpha \rrbracket \triangleright \llbracket \beta \rrbracket$, computations $(v, \tau_1, u) \in \llbracket \alpha \rrbracket$ and $(u, \tau_2, w) \in \llbracket \beta \rrbracket$ with $\tau = \tau_1 \cdot \tau_2$ exist. Now, we conclude as follows:

1. For (commit), assume $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A$. If $\tau_2 = \epsilon$, (commit) by the assumption because $(v, \tau, \perp) \in \llbracket \alpha \rrbracket_\perp$. If $\tau_2 \neq \epsilon$, then $\{v \cdot \tau' \mid \tau' \preceq \tau_1\} \models A$. Hence, $u \cdot \tau_1 \models [\beta]_{\{A, C\}} \psi$ by (post). By Lemma 8, $\{u \cdot \tau' \mid \tau' \prec \tau_1 \cdot \tau_2\} \models A$ which implies $\{u \cdot \tau_1 \cdot \tau' \mid \tau' \prec \tau_2\} \models A$. By $u \cdot \tau_1 \models [\beta]_{\{A, C\}} \psi$ and $(u, \tau_2, w) \in \llbracket \beta \rrbracket$, we obtain $u \cdot \tau_1 \cdot \tau_2 \models C$. Using Lemma 8 again, $v \cdot \tau_1 \cdot \tau_2 \models C$.
2. For (post), assume $\{v \cdot \tau' \mid \tau' \preceq \tau\} \models A$ and $w \neq \perp$. Then $u \cdot \tau_1 \models [\beta]_{\{A, C\}} \psi$ as above. Finally, $w \cdot \tau \models \psi$ because $\{u \cdot \tau_1 \cdot \tau' \mid \tau' \preceq \tau_2\} \models A$ by Lemma 8.

$\llbracket \cup \rrbracket_{\mathcal{A}\mathcal{C}}$: The axiom follows directly from the semantics $\llbracket \alpha \cup \beta \rrbracket = \llbracket \alpha \rrbracket \cup \llbracket \beta \rrbracket$ of choice.

$[*]_{\mathcal{AC}}$ Since $\llbracket \alpha^* \rrbracket = \bigcup_{n \in \mathbb{N}} \llbracket \alpha^n \rrbracket$, the formula $[\alpha^*]_{\{A,C\}} \psi \leftrightarrow [\alpha^0]_{\{A,C\}} \psi \wedge [\alpha; \alpha^*]_{\{A,C\}} \psi$ is valid. Axiom $[*]_{\mathcal{AC}}$ follows from the formula by axiom $[\cdot]_{\mathcal{AC}}$.

$I_{\mathcal{AC}}$: Let $v \models [\alpha^*]_{\{A,C\}} \psi$. Then $v \models [\alpha^0]_{\{A,C\}} \psi \wedge [\alpha; \alpha^*]_{\{A,C\}} \psi$ by axioms $[*]_{\mathcal{AC}}$ and $[\cdot]_{\mathcal{AC}}$. Since $\llbracket \alpha; \alpha^* \rrbracket = \llbracket \alpha^*; \alpha \rrbracket$ by induction and Lemma 3, $v \models [\alpha^*]_{\{A,C\}} [\alpha]_{\{A,C\}} \psi$ by axiom $[\cdot]_{\mathcal{AC}}$. Finally, $v \models [\alpha^*]_{\{A,T\}} (\psi \rightarrow [\alpha]_{\{A,C\}} \psi)$ by rule $M[\cdot]_{\mathcal{AC}}$.

Conversely, let $v \models [\alpha^0]_{\{A,C\}} \psi \wedge [\alpha^*]_{\{A,T\}} (\psi \rightarrow [\alpha]_{\{A,C\}} \psi)$ and $(v, \tau, w) \in \llbracket \alpha^* \rrbracket$. Since $(v, \tau, w) \in \llbracket \alpha^n \rrbracket$ for some $n \in \mathbb{N}$, we proceed by induction on n :

1. $n = 0$, then (commit) holds by $v \models [\alpha^0]_{\{A,C\}} \psi$ and axiom $W[\cdot]_{\mathcal{AC}}$ since $\tau = \epsilon$. For (post), additionally assume $w \neq \perp$ and observe that $v = w$ since $\alpha^0 \equiv ?T$ such that $w \models \psi$.
2. $n > 0$, then $(v, \tau, w) \in \llbracket \alpha^n \rrbracket = \llbracket \alpha; \alpha^{n-1} \rrbracket$, which, by Lemma 3, equals $\llbracket \alpha^{n-1}; \alpha \rrbracket = \llbracket \alpha^{n-1} \rrbracket_{\perp} \cup \llbracket \alpha^{n-1} \rrbracket_{\triangleright} [\alpha]$. In case $\llbracket \alpha^{n-1} \rrbracket_{\perp} \subseteq \llbracket \alpha^{n-1} \rrbracket$, (commit) and (post) hold by IH. Otherwise, if $(v, \tau, w) \in \llbracket \alpha^{n-1} \rrbracket_{\triangleright} [\alpha]$, computations $(v, \tau_1, u) \in \llbracket \alpha^{n-1} \rrbracket$ and $(u, \tau_2, w) \in \llbracket \alpha \rrbracket$ exist with $\tau = \tau_1 \cdot \tau_2$.
 - (a) If $\tau_2 = \epsilon$, (commit) holds by IH since $(v, \tau, \perp) \in \llbracket \alpha^{n-1} \rrbracket$. If $\tau_2 \neq \epsilon$, assume $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A$, which implies $\{v \cdot \tau' \mid \tau' \preceq \tau_1\} \models A$. Thus, $u \cdot \tau_1 \models \psi \rightarrow [\alpha]_{\{A,C\}} \psi$ by assumption. Since $u \cdot \tau_1 \models \psi$ by IH and (post), we obtain $u \cdot \tau_1 \models [\alpha]_{\{A,C\}} \psi$. By Lemma 8 and assumption $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A$, we obtain $\{u \cdot \tau_1 \cdot \tau' \mid \tau' \prec \tau_2\} \models A$. Thus, $u \cdot \tau_1 \cdot \tau_2 \models C$, which implies $v \cdot \tau \models C$ by Lemma 8 again.
 - (b) For (post), let $w \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau\} \models A$. Then $\{v \cdot \tau' \mid \tau' \preceq \tau_1\} \models A$ and $\{v \cdot \tau_1 \cdot \tau' \mid \tau' \preceq \tau_2\} \models A$. As in case (commit), we obtain $u \cdot \tau_1 \models [\alpha]_{\{A,C\}} \psi$. Now, Lemma 8 implies $\{u \cdot \tau_1 \cdot \tau' \mid \tau' \preceq \tau_2\} \models A$ such that $w \cdot \tau \models \psi$.

$[\cdot]_{\perp} \mathcal{AC}$: Let $v \models [\alpha]_{\{A,C\}} \psi$ and $(v, \tau, w) \in \llbracket \alpha \parallel \beta \rrbracket$. Then $(v, \tau \downarrow \alpha, w_{\alpha}) \in \llbracket \alpha \rrbracket$ with $w = w_{\alpha} \oplus w_{\beta}$ for some $w_{\beta} \in \mathcal{S}_{\perp}$. For (commit), assume $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A$. If $\tau'_{\alpha} \prec \tau \downarrow \alpha$, then $\tau' \prec \tau$ exists such that $\tau'_{\alpha} = \tau' \downarrow \alpha$. Thus, $(v, \tau', \perp) \in \llbracket \alpha \parallel \beta \rrbracket$ and $(v, \tau'_{\alpha}, \perp) \in \llbracket \alpha \rrbracket$ by prefix-closedness (Proposition 2). Hence, $\{v \cdot \tau' \mid \tau' \prec \tau \downarrow \alpha\} \models A$ by Lemma 9 since β does not interfere with $[\alpha]_{\{A,C\}} \psi$. Therefore, $v \cdot (\tau \downarrow \alpha) \models C$ by $v \models [\alpha]_{\{A,C\}} \psi$. Now, we obtain $v \cdot \tau \models C$ using Lemma 9 again. For (post), assume $w \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau\} \models A$. Then $w_{\alpha} \neq \perp$ by the definition of \oplus in Section 2.2 and $\{v \cdot \tau' \mid \tau' \preceq \tau \downarrow \alpha\} \models A$ by Lemma 9 as above. Hence, $w_{\alpha} \cdot (\tau \downarrow \alpha) \models \psi$ by $v \models [\alpha]_{\{A,C\}} \psi$. Finally, $w \cdot \tau \models \psi$ by Lemma 9.

$[\cdot]_{T,T}$: Let $v \models [\alpha] \psi$ and $(v, \tau, w) \in \llbracket \alpha \rrbracket$. Then (commit) holds trivially. For (post), assume $w \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau\} \models T$. Then $w \cdot \tau \models \psi$ by assumption. Conversely, let $v \models [\alpha]_{\{T,T\}} \psi$ and $(v, \tau, w) \in \llbracket \alpha \rrbracket$ with $w \neq \perp$. Then $w \cdot \tau \models \psi$ holds by (post) since $\{v \cdot \tau' \mid \tau' \preceq \tau\} \models T$ holds trivially.

$[\cdot]_{\mathcal{AC}} \wedge$: The implication $[\alpha]_{\{A,C_1\}} \psi_1 \wedge [\alpha]_{\{A,C_2\}} \psi_2 \rightarrow [\alpha]_{\{A,C_1 \wedge C_2\}} (\psi_1 \wedge \psi_2)$ can be easily shown by the semantics. The converse implication derives by rule $M[\cdot]_{\mathcal{AC}}$.

$[\text{ch}!]_{\mathcal{AC}}$: Let $v \models [\text{ch}(h)! \theta]_{\{A,C\}} \psi$. Then $v \models C$ by $W[\cdot]_{\mathcal{AC}}$. Now, assume $v \models A$ and let $(v, \tau, w) \in \llbracket \text{ch}(h)! \theta \rrbracket$ with $w \neq \perp$. Since $v \models A$ and $|\tau| = 1$, we have $\{v \cdot \tau' \mid \tau' \prec \tau\} \models A$. Thus, $v \cdot \tau \models C$ by (commit), which implies $w \cdot \tau \models C$ by

Lemma 8. For $A \rightarrow \psi$, assume $w \cdot \tau \vDash A$. Then $v \cdot \tau \vDash A$ by Lemma 8 such that $\{v \cdot \tau' \mid \tau' \preceq \tau\} \vDash A$. Hence, $w \cdot \tau \vDash \psi$ by (post).

Conversely, let $v \vDash \Phi([\text{ch}(h)!\theta]\Phi(\psi))$, where $\Phi(\phi) \equiv C \wedge (A \rightarrow \phi)$ and $(v, \tau, w) \in \llbracket \text{ch}(h)!\theta \rrbracket$. For (commit), assume $\{v \cdot \tau' \mid \tau' \prec \tau\} \vDash A$. If $\tau = \epsilon$, we obtain $v \vDash C$ by assumption. If $\tau \neq \epsilon$, then $v \vDash A$ such that $v \vDash [\text{ch}(h)!\theta]\Phi(\psi)$. W.l.o.g. $w \neq \perp$ since $\text{ch}(h)!\theta$ has a terminating computation from any v . Hence, $w \cdot \tau \vDash \Phi(\psi)$ such that $w \cdot \tau \vDash C$. By Lemma 8, we obtain $v \cdot \tau \vDash C$. For (post), assume $w \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau\} \vDash A$. As above $w \cdot \tau \vDash \Phi(\psi)$ such that $w \cdot \tau \vDash A \rightarrow \psi$. Since $v \cdot \tau \vDash A$ by assumption, which implies $w \cdot \tau \vDash A$ by Lemma 8, we obtain $w \cdot \tau \vDash \psi$.

[ch!]: Let $v \vDash \forall h_0 (h_0 = h \cdot \langle \text{ch}, \theta, \mu \rangle \rightarrow \psi(h_0))$ and $(v, \tau, w) \in \llbracket \text{ch}(h)!\theta \rrbracket$ with $w \neq \perp$. Instantiating $\forall h_0$ with $\tau_0 = \llbracket h \cdot \langle \text{ch}, \theta, \mu \rangle \rrbracket v$ to $u = v_{h_0}^{\tau_0}$, we have $u \vDash h_0 = h \cdot \langle \text{ch}, \theta, \mu \rangle$. Therefore, $u \vDash \psi(h_0)$. By substitution, $u_{h_0}^{\llbracket h_0 \rrbracket u} \vDash \psi(h)$. Since $h_0 \notin FV(\theta)$, we have $\llbracket h \cdot \langle \text{ch}, \theta, \mu \rangle \rrbracket u = \llbracket h \cdot \langle \text{ch}, \theta, \mu \rangle \rrbracket v$ by coincidence (Lemma 6). Moreover, $\llbracket h_0 \rrbracket u = \llbracket h \cdot \langle \text{ch}, \theta, \mu \rangle \rrbracket u$ such that $u_{h_0}^{\llbracket h \cdot \langle \text{ch}, \theta, \mu \rangle \rrbracket v} \vDash \psi(h)$, which implies $v_{h_0}^{\llbracket h \cdot \langle \text{ch}, \theta, \mu \rangle \rrbracket v} \vDash \psi(h)$ by coincidence because $h_0 \notin FV(\psi(h))$. Now, observe that $\tau = \langle h, \text{ch}, \llbracket \theta \rrbracket v, v(\mu) \rangle$ such that $v \cdot \tau = v_h^{\llbracket h \cdot \langle \text{ch}, \theta, \mu \rangle \rrbracket v}$. Finally, $w \cdot \tau \vDash \psi(h)$ because $w = v$.

Conversely, let $v \vDash [\text{ch}(h)!\theta]\psi(h)$. To prove the quantifier let $\tau_0 \in \mathcal{T}$ and assume $v_{h_0}^{\tau_0} \vDash h_0 = h \cdot \langle \text{ch}, \theta, \mu \rangle$. Now, observe that $(v, \tau, v) \in \llbracket \text{ch}(h)!\theta \rrbracket$ with $\tau = \langle h, \text{ch}, \llbracket \theta \rrbracket v, v(\mu) \rangle$ such that $v \cdot \tau \vDash \psi(h)$. Since $v \cdot \tau = v_h^{\llbracket h \cdot \langle \text{ch}, \theta, \mu \rangle \rrbracket v}$ and $\llbracket h \cdot \langle \text{ch}, \theta, \mu \rangle \rrbracket v = \llbracket h_0 \rrbracket v = \tau_0$, we obtain $v_{h_0}^{\tau_0} \vDash \psi(h_0)$ by substitution.

[ch?]_{AC}: First, observe that $\llbracket \text{ch}(h)?x \rrbracket = \llbracket x := * \rrbracket \triangleright \llbracket \text{ch}(h)!x \rrbracket$.

Now, let $v \vDash [\text{ch}(h)?x]_{\{A, C\}} \psi$ and w.l.o.g. $(v, \epsilon, u) \in \llbracket x := * \rrbracket$ with $u \neq \perp$. Futher, let $(u, \tau, w) \in \llbracket \text{ch}(h)!x \rrbracket$. Then $(v, \tau, w) \in \llbracket x := * \rrbracket \triangleright \llbracket \text{ch}(h)!x \rrbracket = \llbracket \text{ch}(h)?x \rrbracket$. For (commit), assume $\{u \cdot \tau' \mid \tau' \prec \tau\} \vDash A$. Since $[\text{ch}(h)?x]_{\{A, C\}} \psi$ is well-formed, we have $(FV(A) \cup FV(C)) \cap BV(\text{ch}(h)?x) \subseteq V_{\mathcal{T}}$. Since $x \in BV(\text{ch}(h)?x)$ and $BV(x := *) = \{x\}$, we have $v = u$ on $FV(A) \cup FV(C)$. By coincide (Lemma 6), we obtain $\{v \cdot \tau' \mid \tau' \prec \tau\} \vDash A$ such that $v \cdot \tau \vDash C$ by premise, which implies $u \cdot \tau \vDash C$ by coincidence again. For (post), assume $w \neq \perp$ and $\{u \cdot \tau' \mid \tau' \preceq \tau\} \vDash A$. Then $\{v \cdot \tau' \mid \tau' \preceq \tau\} \vDash A$ as above. By premise, $w \cdot \tau \vDash \psi$.

Conversely, let $v \vDash [x := *][\text{ch}(h)!x]_{\{A, C\}} \psi$ and $(v, \tau, w) \in \llbracket \text{ch}(h)?x \rrbracket$. Then $(v, \epsilon, u) \in \llbracket x := * \rrbracket$ and $(u, \tau, w) \in \llbracket \text{ch}(h)!x \rrbracket$ exist. By premise, we obtain $u \vDash \phi$, where $\phi \equiv [\text{ch}(h)!x]_{\{A, C\}} \psi$. Now, we have to prove $v \vDash [\text{ch}(h)?x]_{\{A, C\}} \psi$. Therefore, assume $\{v \cdot \tau' \mid \tau' \prec \tau\} \vDash A$ for (commit). As in the converse direction, $\{u \cdot \tau' \mid \tau' \prec \tau\} \vDash A$ sucht that $u \cdot \tau \vDash C$ by $u \vDash \phi$, which again implies $v \cdot \tau \vDash C$. For (post), assume $w \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau\}$. Finally, $w \cdot \tau \vDash \psi$ by $u \vDash \phi$.

M[·]_{AC}: Let $v \vDash [\alpha]_{\{A_1, C_1\}} \psi$ and $(v, \tau, w) \in \llbracket \alpha \rrbracket$. For (commit), assume $\{v \cdot \tau' \mid \tau' \prec \tau\} \vDash A_2$. Then $\{v \cdot \tau' \mid \tau' \prec \tau\} \vDash A_1$, which implies $v \cdot \tau \vDash C_1$ by assumption. Finally, $v \cdot \tau \vDash C_2$ by premise $C_1 \rightarrow C_2$. For (post), assume $w \neq \perp$ and $\{v \cdot \tau' \mid \tau' \preceq \tau\} \vDash A_2$. Then $\{v \cdot \tau' \mid \tau' \preceq \tau\} \vDash A_1$ by premise $A_2 \rightarrow A_1$. Thus, $w \cdot \tau \vDash \psi_1$ by assumption. Finally, $w \cdot \tau \vDash \psi_2$ by premise $\psi_1 \rightarrow \psi_2$.

G_{AC}: Let the premise $C \wedge \psi$ be valid. To prove validity of $[\alpha]_{\{A,C\}}\psi$, let v be an arbitrary state and $(v, \tau, w) \in \llbracket \alpha \rrbracket$. Condition (commit) holds since $v \cdot \tau \models C$ by validity of C . For (post), assume $w \neq \perp$. Then $w \cdot \tau \models \psi$ since ψ is valid.

[μ]: Let $ode \equiv \{x' = \theta \ \& \ \chi\}$. Now, $(v, \tau, w) \in \llbracket \{\mu' = 1, ode\} \rrbracket$ iff a solution $\varphi : [0, s] \rightarrow \mathcal{S}$ from v to w exists such that $\varphi(\zeta) \models \mu' = 1 \wedge \mu' = 1 \wedge x' = \theta$ and $v = \varphi(\zeta)$ on $\{x, \mu\}^G$ for $\zeta \in [0, s]$ iff, since $\mu' = 1 \wedge \mu' = 1 \leftrightarrow \mu' = 1$ at all $\varphi(\zeta)$, we have $(v, \tau, w) \in \llbracket ode \rrbracket$. Thus, $v \models [\{\mu' = 1, ode\}]\psi$ iff $v \models [ode]\psi$. \square

Corollary 1. *The axioms and rules in Fig. 5 derive syntactically. Additionally, we derive the following rule **inv_{AC}** in preparation of the proof of rule **ind_{AC}**:*

$$\text{inv}_{AC} \frac{I \vdash [\alpha]_{\{A,C\}}\psi}{I \vdash [\alpha^*]_{\{A,C\}}\psi}$$

Proof (Corollary 1). We prove the axioms and rules by derivation in our calculus. For rule **CG**, see Fig. 16, for rule **[$\llbracket _ \rrbracket_{AC}R$]**, see Fig. 17 for rule **inv_{AC}**, see Fig. 18, for rule **ind_{AC}**, see Fig. 19, and for rule **[$!\theta$]_{AC}R**, see Fig. 20. \square

$$\begin{array}{c} = \\ \rightarrow L \frac{\frac{*}{\vdash h \cdot te = h \cdot te, \Phi(h)} \quad \frac{*}{\Phi(h) \vdash \Phi(h)} \text{Id}}{h \cdot te = h \cdot te \rightarrow \Phi(h) \vdash \Phi(h)} \\ \forall L \frac{\forall h_0 (h_0 = h \cdot te \rightarrow \Phi(h_0)) \vdash \Phi(h)}{\Gamma \vdash \forall h_0 (h_0 = h \cdot te \rightarrow \Phi(h_0)), \Delta} \\ [\text{ch!}] \frac{[\text{ch}(h)!\theta]\Phi(h) \vdash \Phi(h)}{\Gamma, [\text{ch}(h)!\theta]\Phi(h) \vdash \Phi(h), \Delta} \\ \text{WL, WR} \frac{\Gamma, [\text{ch}(h)!\theta]\Phi(h) \vdash \Phi(h), \Delta}{\Gamma \vdash \Phi(h), \Delta} \\ \end{array} \quad \begin{array}{c} \rightarrow R \frac{\Gamma, h_0 = h \cdot te \vdash \Phi(h_0), \Delta}{\Gamma \vdash h_0 = h \cdot te \rightarrow \Phi(h_0), \Delta} \\ \forall R \frac{\Gamma \vdash \forall h_0 (h_0 = h \cdot te \rightarrow \Phi(h_0)), \Delta}{\Gamma \vdash [\text{ch}(h)!\theta]\Phi(h), \Delta} \\ [\text{ch!}] \frac{\Gamma \vdash [\text{ch}(h)!\theta]\Phi(h), \Delta}{\Gamma \vdash [\text{ch}(h)!\theta]\Phi(h), \Phi(h), \Delta} \\ \text{WR} \\ \text{Cut} \end{array}$$

Fig. 16: Derivation of rule **CG**. In the proof, te abbreviates $\langle \text{ch}, \theta, \mu \rangle$ and $\Phi(h)$ abbreviates $[\alpha(h)]\psi$.

$$\begin{array}{c} \text{WL, WR} \frac{\frac{\frac{\vdash K}{\vdash \text{Cl}_{\forall}K}}{\Gamma \vdash \text{Cl}_{\forall}K, \Delta} \quad \frac{\frac{\frac{\Gamma \vdash [\alpha_j]_{\{A_j, C_j\}}\psi_j, \Delta \quad (\text{for } j = 1, 2)}{\Gamma \vdash [\alpha_1 \parallel \alpha_2]_{\{A_j, C_j\}}\psi_j, \Delta} \quad (\text{for } j = 1, 2)}{\Gamma \vdash [\alpha_1 \parallel \alpha_2]_{\{A_1 \wedge A_2, C_1 \wedge C_2\}}(\psi_1 \wedge \psi_2), \Delta} \quad \llbracket _ \rrbracket_{AC} \\ \quad \quad \quad \triangleleft \quad \quad \quad \llbracket _ \rrbracket_{AC} \\ \quad \quad \quad \text{M}[_]_{AC}}{\Gamma \vdash [\alpha_1 \parallel \alpha_2]_{\{A_1 \wedge A_2, C_1 \wedge C_2\}}(\psi_1 \wedge \psi_2), \Delta} \quad \llbracket _ \rrbracket_{AC} \wedge \\ \wedge R \frac{\Gamma \vdash \text{Cl}_{\forall}K \wedge [\alpha_1 \parallel \alpha_2]_{\{A_1 \wedge A_2, C_1 \wedge C_2\}}(\psi_1 \wedge \psi_2), \Delta}{\Gamma \vdash [\alpha_1 \parallel \alpha_2]_{\{A, C_1 \wedge C_2\}}(\psi_1 \wedge \psi_2), \Delta} \quad \llbracket _ \rrbracket_{AC} \kappa \end{array}$$

Fig. 17: Derivation of rule **[$\llbracket _ \rrbracket_{AC}R$]**. For $j = 1, 2$, the open premise $A_1 \wedge A_2 \rightarrow A_j$ marked by \triangleleft obviously holds. Axiom **[$\llbracket _ \rrbracket_{AC}$]** can be applied since α_{3-j} does not interfere with $[\alpha_j]_{\{A_j, C_j\}}\psi_j$ by premise.

$$\begin{array}{c}
 \begin{array}{c}
 \text{Id} \frac{*}{I, C, [\alpha]_{\{A, C\}} I \vdash C} \\
 \wedge L \frac{I, C \wedge [\alpha]_{\{A, C\}} I \vdash C}{I, C, [\alpha]_{\{A, C\}} I \vdash C} \\
 \text{W}[\cdot]_{\mathcal{AC}} \frac{I, [\alpha]_{\{A, C\}} I \vdash C}{I \vdash C} \\
 \text{Cut} \frac{I \vdash C \quad I \vdash [\alpha]_{\{A, C\}} I}{I \vdash C}
 \end{array}
 \qquad
 \begin{array}{c}
 \text{Id} \frac{*}{I, A, \top \vdash I} \\
 \rightarrow R \frac{I, A \vdash \top \rightarrow I}{I, A \vdash \top \rightarrow I} \\
 [\text{?}] \frac{I, A \vdash [\alpha^0] I}{I, A \vdash [\alpha^0] I} \\
 \rightarrow R \frac{I \vdash A \rightarrow [\alpha^0] I}{I \vdash A \rightarrow [\alpha^0] I}
 \end{array}
 \\
 \\
 \begin{array}{c}
 \wedge R \frac{I \vdash C \wedge (A \rightarrow [\alpha^0] I)}{I \vdash C \wedge (A \rightarrow [\alpha^0] I)} \\
 [\epsilon]_{\mathcal{AC}} \frac{I \vdash [\alpha^0]_{\{A, C\}} I \quad I \vdash [\alpha]_{\{A, C\}} I}{I \vdash [\alpha^0]_{\{A, C\}} I \wedge [\alpha]_{\{A, C\}} I} \\
 \rightarrow R \frac{I \vdash [\alpha]_{\{A, C\}} I}{\vdash \top} \\
 \rightarrow R \frac{I \vdash [\alpha]_{\{A, C\}} I}{\vdash I \rightarrow [\alpha]_{\{A, C\}} I} \\
 \wedge R \frac{\vdash \top \wedge (I \rightarrow [\alpha]_{\{A, C\}} I)}{\vdash \top \wedge (I \rightarrow [\alpha]_{\{A, C\}} I)} \\
 \text{G}_{\mathcal{AC}} \frac{I \vdash [\alpha^*]_{\{A, T\}} (I \rightarrow [\alpha]_{\{A, C\}} I)}{I \vdash [\alpha^*]_{\{A, T\}} (I \rightarrow [\alpha]_{\{A, C\}} I)} \\
 \wedge R \frac{I \vdash [\alpha^0]_{\{A, C\}} I \wedge [\alpha^*]_{\{A, T\}} (I \rightarrow [\alpha]_{\{A, C\}} I)}{I \vdash [\alpha^0]_{\{A, C\}} I \wedge [\alpha^*]_{\{A, T\}} (I \rightarrow [\alpha]_{\{A, C\}} I)} \\
 \text{I}_{\mathcal{AC}} \frac{I \vdash [\alpha^*]_{\{A, C\}} I}{I \vdash [\alpha^*]_{\{A, C\}} I}
 \end{array}
 \end{array}$$

 Fig. 18: Derivation of rule $\text{inv}_{\mathcal{AC}}$

$$\begin{array}{c}
 \begin{array}{c}
 \wedge L \frac{C, I \vdash [\alpha]_{\{A, C\}} I}{I_C \vdash [\alpha]_{\{A, C\}} I} \\
 \text{M}[\cdot]_{\mathcal{AC}} \frac{I_C \vdash [\alpha]_{\{A, C\}} I}{I_C \vdash [\alpha]_{\{A, C\}} I_C} \\
 \text{inv}_{\mathcal{AC}} \frac{I_C \vdash [\alpha^*]_{\{A, C\}} I_C}{I_C \vdash [\alpha^*]_{\{A, C\}} I_C} \\
 \rightarrow R \frac{\Gamma \vdash I_C \rightarrow [\alpha^*]_{\{A, C\}} I_C, \Delta}{\Gamma \vdash I_C \rightarrow [\alpha^*]_{\{A, C\}} I_C, \Delta}
 \end{array}
 \qquad
 \begin{array}{c}
 \text{Id} \frac{*}{I, C \vdash C} \\
 \wedge L \frac{C, A, I \vdash \psi}{I_C, A \vdash \psi} \\
 \wedge R \frac{I_C \vdash C \quad I_C \vdash A \rightarrow \psi}{I_C \vdash (C \wedge (A \rightarrow \psi))} \\
 \text{M}[\cdot]_{\mathcal{AC}} \frac{I_C \vdash (C \wedge (A \rightarrow \psi))}{[\alpha^*]_{\{A, C\}} I_C \vdash [\alpha^*]_{\{A, C\}} (C \wedge (A \rightarrow \psi))} \\
 \text{W}[\cdot]_{\mathcal{AC}} \frac{[\alpha^*]_{\{A, C\}} I_C \vdash [\alpha^*]_{\{A, C\}} (C \wedge (A \rightarrow \psi))}{[\alpha^*]_{\{A, C\}} I_C \vdash [\alpha^*]_{\{A, C\}} \psi} \\
 \rightarrow L \frac{\Gamma \vdash I_C, \Delta \quad [\alpha^*]_{\{A, C\}} I_C \vdash [\alpha^*]_{\{A, C\}} \psi, \Delta}{\Gamma, I_C \rightarrow [\alpha^*]_{\{A, C\}} I_C \vdash [\alpha^*]_{\{A, C\}} \psi, \Delta} \\
 \text{Cut} \frac{\Gamma \vdash I_C \rightarrow [\alpha^*]_{\{A, C\}} I_C, \Delta \quad \Gamma, I_C \rightarrow [\alpha^*]_{\{A, C\}} I_C \vdash [\alpha^*]_{\{A, C\}} \psi, \Delta}{\Gamma \vdash [\alpha^*]_{\{A, C\}} \psi, \Delta}
 \end{array}
 \end{array}$$

 Fig. 19: Derivation of rule $\text{ind}_{\mathcal{AC}}$. In the proof, we silently apply weakening several times. The formula $I_C \equiv I \wedge C$ combines the invariant with the commitment C . Moreover, Ψ is short for $C \wedge (A \rightarrow \psi)$.

$$\begin{array}{c}
 \begin{array}{c}
 \rightarrow R \frac{\Gamma, A(h), H_0 \vdash C(h_0), \Delta \quad \Gamma, A(h), H_0, A(h_0) \vdash \psi(h_0), \Delta}{\Gamma, A(h), H_0 \vdash A(h_0) \rightarrow \psi(h_0), \Delta} \\
 \wedge R \frac{\Gamma, A(h), H_0 \vdash C(h_0), \Delta \quad \Gamma, A(h), H_0, A(h_0) \vdash \psi(h_0), \Delta}{\Gamma, A(h), H_0 \vdash C(h_0) \wedge (A(h_0) \rightarrow \psi(h_0)), \Delta} \\
 \rightarrow R \frac{\Gamma, A(h), H_0 \vdash C(h_0) \wedge (A(h_0) \rightarrow \psi(h_0)), \Delta}{\Gamma, A(h) \vdash H_0 \rightarrow (C(h_0) \wedge (A(h_0) \rightarrow \psi(h_0))), \Delta} \\
 \forall R \frac{\Gamma, A(h) \vdash \forall h_0 (H_0 \rightarrow (C(h_0) \wedge (A(h_0) \rightarrow \psi(h_0)))) \wedge \Delta}{\Gamma, A(h) \vdash \forall h_0 (H_0 \rightarrow (C(h_0) \wedge (A(h_0) \rightarrow \psi(h_0)))) \wedge \Delta} \\
 [\text{ch}] \frac{\Gamma, A(h) \vdash [\text{ch}(h)! \theta] (C(h) \wedge (A(h) \rightarrow \psi(h))), \Delta}{\Gamma, A(h) \vdash [\text{ch}(h)! \theta] (C(h) \wedge (A(h) \rightarrow \psi(h))), \Delta} \\
 \rightarrow R \frac{\Gamma \vdash C(h), \Delta \quad \Gamma \vdash A(h) \rightarrow [\text{ch}(h)! \theta] (C(h) \wedge (A(h) \rightarrow \psi(h))), \Delta}{\Gamma \vdash C(h) \wedge (A(h) \rightarrow [\text{ch}(h)! \theta] (C(h) \wedge (A(h) \rightarrow \psi(h))), \Delta} \\
 \wedge R \frac{\Gamma \vdash C(h) \wedge (A(h) \rightarrow [\text{ch}(h)! \theta] (C(h) \wedge (A(h) \rightarrow \psi(h))), \Delta}{\Gamma \vdash [\text{ch}(h)! \theta]_{\{A(h), C(h)\}} \psi(h), \Delta} \\
 [\text{ch}]_{\mathcal{AC}}
 \end{array}
 \end{array}$$

 Fig. 20: Derivation of rule $[\text{!}\theta]_{\mathcal{AC}R}$. In the proof, H_0 is short for $h \cdot \langle \text{ch}, \theta, \mu \rangle$.

D Details of the Example

We continue the proof of the example from Section 3 and provide the remaining derivations:

\triangleright Fig. 22	
$C, L, \chi_{vel}(v_0) \vdash [\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h)$	
$C, L, \chi_{vel}(v_0) \vdash [\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h \cdot \langle \text{vel}, v_0, \mu \rangle)$	$\downarrow \notin$
$C, L, \chi_{vel}(v_0), h_{\text{vel}} = h \cdot \langle \text{vel}, v_0, \mu \rangle$	$=R$
$\vdash [\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h_{\text{vel}})$	$\triangleright \text{Id} \quad \triangleright \text{Fig. 23}$
$C, L, \chi_{vel}(v_0) \vdash [\text{vel}(h)!v_0]_{\{T,C\}}[\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h)$	$\triangleright \text{Fig. 24}$
$C, L, \chi_{vel}(v_0) \vdash [\text{vel}(h)!v_0 \cup \text{skip}]_{\{T,C\}}[\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h)$	$[\! \theta]\mathcal{A}R$
$C, L \vdash [?\chi_{vel}(v_0)][\text{vel}(h)!v_0 \cup \text{skip}]_{\{T,C\}}[\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h)$	$[\cup]\mathcal{A}$
$C, L \vdash [v_l := *][?\chi_{vel}(v_l)][\text{vel}(h)!v_l \cup \text{skip}]_{\{T,C\}}[\mathbf{plant}_l(v_l)]L(v_l, x_l, w, \mu, h)$	$[?], \rightarrow R$
$C, L \vdash [v_l := *; ?\chi_{vel}(v_l)][\text{vel}(h)!v_l \cup \text{skip}]_{\{T,C\}}[\mathbf{plant}_l(v_l)]L(v_l, x_l, w, \mu, h)$	$[:*], \forall R$
$C, L \vdash C \wedge [v_l := *; ?\chi_{vel}(v_l)][\text{vel}(h)!v_l \cup \text{skip}]_{\{T,C\}}[\mathbf{plant}_l(v_l)]L(v_l, x_l, w, \mu, h)$	$[i]$
$C, L \vdash [v_l := *; ?\chi_{vel}(v_l)]_{\{T,C\}}[\text{vel}(h)!v_l \cup \text{skip}]_{\{T,C\}}[\mathbf{plant}_l(v_l)]L(v_l, x_l, w, \mu, h)$	$\triangleright \text{Id}$
$C, L \vdash C \wedge [v_l := *; ?\chi_{vel}(v_l)]_{\{T,C\}}[\text{vel}(h)!v_l \cup \text{skip}]_{\{T,C\}}[\mathbf{plant}_l(v_l)]L(v_l, x_l, w, \mu, h)$	$\wedge R$
$C, L \vdash [v_l := *; ?\chi_{vel}(v_l)]_{\{T,C\}}[\text{vel}(h)!v_l \cup \text{skip}]_{\{T,C\}}[\mathbf{plant}_l(v_l)]L(v_l, x_l, w, \mu, h)$	$[\epsilon]\mathcal{A}$
$C, L \vdash [\text{noti}(h)]_{\{T,C\}}[\mathbf{plant}_l(v_l)]L(v_l, x_l, w, \mu, h)$	$[i]\mathcal{A}$
$\triangleright \text{Fig. 25}$	
$\text{Execution by } [i]\mathcal{A}, [\cup]\mathcal{A}, [\epsilon]\mathcal{A}, W[\mathcal{A}], \text{ and } \wedge R$	
$C, L \vdash [(\text{noti}(h) \cup \text{updt}(h)); \mathbf{plant}_l(v_l)]_{\{T,C\}}L(v_l, x_l, w, \mu, h)$	$\triangleright_1 \text{TA}, \mathbb{R} \quad \triangleright_2 \mathbb{R}$
$\Gamma \vdash [\text{leader}(h)]_{\{T,C\}}\psi_l$	$\text{ind}_{\mathcal{A}}$

Fig. 21: Continues the derivation from Fig. 7. With Γ we abbreviate the list $\varphi, h_1 = h_0 \cdot \langle \text{vel}, 0, \mu \rangle, h = h_1 \cdot \langle \text{pos}, x_l, \mu \rangle$ of formulas. The induction uses invariant $L \equiv \text{val}(h \downarrow \text{pos}) \leq x_l \wedge v_l \geq 0 \wedge w = \Delta(\mu, h \downarrow \text{pos}) \leq \epsilon$. The induction base $\Gamma \vdash L$ (\triangleright_1) closes by trace algebra **TA** and real arithmetic \mathbb{R} . Postcondition $L \vdash \psi_l$ (\triangleright_2) holds by \mathbb{R} .

$*$	\mathbb{R}
$L, \chi_{vel}(v_0), t \geq 0, w + t \leq \epsilon$	
$\vdash w + t = \Delta(\mu + t, h) \leq \epsilon \wedge v_0 \geq 0 \wedge \text{val}(h) \leq x_l + t \cdot v_0$	$\forall L$
$L, \chi_{vel}(v_0), t \geq 0, \mathcal{E} \vdash L(v_0, x_l + t \cdot v_0, w + t, \mu + t, h)$	$\rightarrow R, WL$
$C, L, \chi_{vel}(v_0), t \geq 0 \vdash \mathcal{E} \rightarrow L(v_0, x_l + t \cdot v_0, w + t, \mu + t, h)$	$\forall R$
$C, L, \chi_{vel}(v_0) \vdash \forall t \geq 0 (\mathcal{E} \rightarrow L(v_0, x_l + t \cdot v_0, w + t, \mu + t, h))$	$['], [:=]$
$C, L, \chi_{vel}(v_0) \vdash [\{\mu' = 1, w' = 1, x_l' = v_0 \ \& \ w \leq \epsilon\}]L(v_0, x_l, w, \mu, h)$	$[\mu]$
$C, L, \chi_{vel}(v_0) \vdash [\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h)$	

Fig. 22: Continues the proofs from Fig. 21 and Fig. 24. Thereby \mathcal{E} is short for $\forall 0 \leq \hat{t} \leq t \ w \leq \hat{t} \leq \epsilon$.

$*$ $C, L, \chi_{vel}(v_0) \vdash 0 \leq v_0 \leq V$	$\triangleright \mathbb{N}$ Id
$C, L, \chi_{vel}(v_0) \vdash 0 \leq \mathbf{val}(h_{vel} \cdot \langle \text{out}, v_0, \mu \rangle) \leq V$	val=
$C, L, \chi_{vel}(v_0), h_{vel} = h \cdot \langle \text{out}, v_0, \mu \rangle \vdash 0 \leq \mathbf{val}(h_0) \leq V$	=R

Fig. 23: Continues the proof from Fig. 21

\triangleright Fig. 22	
$C, L, \chi_{vel}(v_0) \vdash [\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h)$	$\triangleright \text{Id}$
$C, L, \chi_{vel}(v_0) \vdash C \wedge [\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h)$	$\wedge R$
$C, L, \chi_{vel}(v_0) \vdash \top \rightarrow C \wedge [\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h)$	$\rightarrow R, \text{WL}$
$C, L, \chi_{vel}(v_0) \vdash [\mathbf{skip}](C \wedge [\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h))$	$\triangleright \text{Id}$ [?]
$C, L, \chi_{vel}(v_0) \vdash C \wedge [\mathbf{skip}](C \wedge [\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h))$	$\wedge R$
$C, L, \chi_{vel}(v_0) \vdash [\mathbf{skip}]_{\{\top, C\}}[\mathbf{plant}_l(v_0)]L(v_0, x_l, w, \mu, h)$	[ϵ] \mathcal{A}

Fig. 24: Continues proof from Fig. 21

$*$ $L, t \geq 0, w + t \leq \epsilon \vdash w + t = \Delta(\mu + t, h_{\text{pos}}) \leq \epsilon \wedge v_l \geq 0 \wedge x_l + t \cdot v_l \leq x_l$	$\triangleright \mathbb{N}$ \mathbb{R}
$L, w = 0, t \geq 0, \mathcal{E} \vdash L(v_l, x_l + t \cdot v_l, w + t, \mu + t, h \cdot \langle \text{pos}, x_l, \mu \rangle)$	val=
$L, H_{\text{pos}}, w = 0, t \geq 0, \mathcal{E} \vdash L(v_l, x_l + t \cdot v_l, w + t, \mu + t, h_{\text{pos}})$	=R
$L, H_{\text{pos}}, w = 0, t \geq 0 \vdash \mathcal{E} \rightarrow L(v_l, x_l + t \cdot v_l, w + t, \mu + t, h_{\text{pos}})$	$\rightarrow R$
$L, H_{\text{pos}}, w = 0 \vdash \forall t \geq 0 (\mathcal{E} \rightarrow L(v_l, x_l + t \cdot v_l, w + t, \mu + t, h_{\text{pos}}))$	$\forall R$
$L, H_{\text{pos}}, w = 0, \vdash [\{\mu' = 1, w' = 1, x'_l = v_l \ \& \ w \leq 0\}]L(v_l, x_l, w, \mu, h_{\text{pos}})$	[']
$L, H_{\text{pos}}, w = 0 \vdash [\mathbf{plant}_l]L(v_l, x_l, w, \mu, h_{\text{pos}})$	[μ]
$C, L, H_{\text{pos}} \vdash [w := 0][\mathbf{plant}_l]L(v_l, x_l, w, \mu, h_{\text{pos}})$	$\triangleright \text{Id}$ [:=], WL
$C, L \vdash [\text{pos}(h)!x_l]_{\{\top, C\}}[w := 0][\mathbf{plant}_l]L(v_l, x_l, w, \mu, h_{\text{pos}})$	[!] θ \mathcal{A} R
$C, L \vdash [\text{pos}(h)!x_l]_{\{\top, C\}}[w := 0]_{\{\top, C\}}[\mathbf{plant}_l]L(v_l, x_l, w, \mu, h_{\text{pos}})$	[ϵ] \mathcal{A} , W] \mathcal{A}
$C, L \vdash [\text{upd}t(h)]_{\{\top, C\}}[\mathbf{plant}_l]L(v_l, x_l, w, \mu, h_{\text{pos}})$	[$\dot{\}$] \mathcal{A}

Fig. 25: Continues the proof from Fig. 21. We abbreviate $h_{\text{pos}} = h \cdot \langle \text{pos}, x_l, \mu \rangle$ by H_{pos} . Thereby \mathcal{E} is short for $\forall 0 \leq \tilde{t} \leq t \ w + \tilde{t} \leq \epsilon$.

$*$ $F, d \leq \epsilon V, t \geq 0 \vdash F(x_f + t \cdot v_f, \mu + t)$	\triangleright \mathbb{R}
$F, A, H_{\text{vel}}, A(h_0), d \leq \epsilon V, t \geq 0 \vdash F(x_f + t \cdot v_f, \mu + t)$	WL
$F, A, H_{\text{vel}}, A(h_0), d \leq \epsilon V \vdash \forall t \geq 0 F(x_f + t \cdot v_f, \mu + t)$	$\forall R$
$F, A, H_{\text{vel}}, A(h_0), d \leq \epsilon V \vdash [\{\mu' = 1, x'_f = v_f\}]F(x_f, \mu)$	[']
$F, A, H_{\text{vel}}, A(h_0), d \leq \epsilon V \vdash [\{x'_f = v_f\}]F(x_f, \mu)$	[μ]

Fig. 26: Continues the proof from Fig. 8. Formula H_{vel} is short for $h_0 = h_{\text{vel}} \cdot \langle \text{vel}, v_{\text{tar}}, \mu \rangle$

$\frac{*}{F, d_0 = m - x_f, m - x_f \leq \epsilon V, 0 \leq v_0 < d_0/\epsilon, t \geq 0 \vdash 0 \leq v_0 \leq d_0/\epsilon}$	\mathbb{R}
$\wedge v_0 \leq V \wedge x_f + t \cdot v_0 + (\epsilon - (\mu + t - \mu))d_0/\epsilon < m$	$\triangleright \mathbb{N}$
$F, d_0 = m - x_f, m - x_f \leq \epsilon V, 0 \leq v_0 < d_0/\epsilon, t \geq 0 \vdash 0 \leq v_0 \leq d_0/\epsilon$	$\downarrow \epsilon, \text{time} =$
$\wedge v_f \leq V \wedge x_f + t \cdot v_0 + (\epsilon - \Delta(\mu + t, h \cdot \langle \text{pos}, m, \mu \rangle))d_0/\epsilon < m$	$\triangleright \mathbb{N}$
$F, d_0 = m - x_f, m - x_f \leq \epsilon V, 0 \leq v_0 < d_0/\epsilon, t \geq 0$	$\downarrow \epsilon, \text{val} =$
$\vdash F(v_0, x_f + t \cdot v_f, d_0, \mu + t, h \cdot \langle \text{pos}, m, \mu \rangle)$	$=R$
$\Gamma_{\text{mes}}, 0 \leq v_0 < d_0/\epsilon, t \geq 0 \vdash F(v_f, x_f + t \cdot v_f, d_0, \mu + t, h_{\text{pos}})$	$\forall R$
$\Gamma_{\text{mes}}, 0 \leq v_0 < d_0/\epsilon \vdash \forall t \geq 0 F(v_f, x_f + t \cdot v_f, d_0, \mu + t, h_{\text{pos}})$	$[\cdot], [:=]$
$\Gamma_{\text{mes}}, 0 \leq v_0 < d_0/\epsilon \vdash [\{\mu' = 1, x'_f = v_0\}]F(v_f, x_f, d_0, \mu, h_{\text{pos}})$	$[\mu]$
$\Gamma_{\text{mes}}, 0 \leq v_0 < d_0/\epsilon \vdash [\{x'_f = v_0\}]F(v_f, x_f, d_0, \mu, h_{\text{pos}})$	$[?], \rightarrow R$
$\Gamma_{\text{mes}} \vdash [?0 \leq v_0 < d_0/\epsilon][\text{plant}_f(v_0)]F(v_f, x_f, d_0, \mu, h_{\text{pos}})$	$[*], \forall R$
$\Gamma_{\text{mes}} \vdash [v_f := *][?0 \leq v_f < d_0/\epsilon][\text{plant}_f]F(v_f, x_f, d_0, \mu, h_{\text{pos}})$	$[i]$
$\Gamma_{\text{mes}} \vdash [v_f := *; ?0 \leq v_f < d_0/\epsilon][\text{plant}_f]F(v_f, x_f, d_0, \mu, h_{\text{pos}})$	$\triangleright \text{Fig. 28}$
$F, H_{\text{pos}}, d_0 = m - x_f \vdash [\text{cond}(d_0)][\text{plant}_f]F(v_f, x_f, d_0, \mu, h_{\text{pos}})$	$[\text{if}]$
$F, H_{\text{pos}} \vdash [d := m - x_f][\text{cond}(d)][\text{plant}_f]F(v_f, x_f, d, \mu, h_{\text{pos}})$	$[:=]$
$F, A(h), H_{\text{pos}} \vdash [d := m - x_f; \text{cond}(d)][\text{plant}_f]F(v_f, x_f, d, \mu, h_{\text{pos}})$	$\triangleright \text{TR}$
$F \vdash [\text{pos}(h)!m]_{\{A(h), T\}}[d := m - x_f; \text{cond}(d)][\text{plant}_f]F(v_f, x_f, d, \mu, h)$	$[\theta]_{\mathcal{A}R}$
$F \vdash [m := *][\text{pos}(h)!m]_{\{A(h), T\}}[d := m - x_f; \text{cond}(d)][\text{plant}_f]F(v_f, x_f, d, \mu, h)$	$[*], \forall R$
$F \vdash [\text{pos}(h)?m]_{\{A(h), T\}}[d := m - x_f; \text{cond}(d)][\text{plant}_f]F(v_f, x_f, d, \mu, h)$	$[\text{ch?}]_{\mathcal{A}}$
$F \vdash [\text{pos}(h)?m]_{\{A(h), T\}}[d := m - x_f; \text{cond}(d)]_{\{A(h), T\}}[\text{plant}_f]F(v_f, x_f, d, \mu, h)$	$[\epsilon]_{\mathcal{A}}, W[\mathcal{A}]$
$F \vdash [\text{dist}(h)]_{\{A(h), T\}}[\text{plant}_f]F$	$[i]_{\mathcal{A}}$

Fig. 27: Continues the proof from Fig. 8. The program cond abbreviates $\text{if}(d \leq \epsilon V) \{v_f := *; ?0 \leq v_f < d/\epsilon\}$. Formula H_{pos} is short for $h_{\text{pos}} = h \cdot \langle \text{pos}, m, \mu \rangle$. Further, Γ_{mes} denotes the formula list $F, H_{\text{pos}}, d_0 = m - x_f, d_0 \leq \epsilon V$.

$F, m - x_f > \epsilon V, t \geq 0 \vdash 0 \leq v_f \leq d_0/\epsilon \wedge v_f \leq V$	\mathbb{R}
$\wedge x_f + t \cdot v_f + (\epsilon - (\mu + t - \mu))d_0/\epsilon < m$	$\triangleright \mathbb{N}$
$F, m - x_f > \epsilon V, t \geq 0 \vdash 0 \leq v_f \leq d_0/\epsilon \wedge v_f \leq V$	$\downarrow \epsilon, \text{time} =$
$\wedge x_f + t \cdot v_f + (\epsilon - \Delta(\mu + t, h \cdot \langle \text{pos}, m, \mu \rangle))d_0/\epsilon < m$	$\triangleright \mathbb{N}$
$F, m - x_f > \epsilon V, t \geq 0 \vdash F(v_f, x_f + t \cdot v_f, d_0, \mu + t, h \cdot \langle \text{pos}, m, \mu \rangle)$	$\downarrow \epsilon, \text{val} =$
$F, H_{\text{pos}}, m - x_f > \epsilon V, t \geq 0 \vdash F(v_f, x_f + t \cdot v_f, d_0, \mu + t, h_{\text{pos}})$	$=R$
$F, H_{\text{pos}}, m - x_f > \epsilon V \vdash \forall t \geq 0 F(v_f, x_f + t \cdot v_f, m - x_f, \mu + t, h_{\text{pos}})$	$\forall R$
$F, H_{\text{pos}}, m - x_f > \epsilon V \vdash [\mu' = 1, x'_f = v_f]F(v_f, x_f, m - x_f, \mu, h_{\text{pos}})$	$[\cdot], [:=]$
$F, H_{\text{pos}}, m - x_f > \epsilon V \vdash [\text{plant}_f]F(v_f, x_f, m - x_f, \mu, h_{\text{pos}})$	$[\mu]$
$F, H_{\text{pos}}, d_0 = m - x_f, d_0 > \epsilon V \vdash [\text{plant}_f]F(v_f, x_f, d_0, \mu, h_{\text{pos}})$	$=L, =R$

Fig. 28: Continues the proof from Fig. 27